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OPTIMAL DIGITAL COMPUTER CONTROL OF NUCLEAR REACTORS

by

Walter C. Lipinski

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Reactor Technology

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OPTIMAL DIGITAL COMPUTER CONTROL OF NUCLEAR REACTORS

by

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Reactor Engineering Division

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ABSTRACT

Lipinski, Walter Charles; *Optimal Digital Computer Control of Nuclear Reactors*; Ph.D., Electrical Engineering Department; Illinois Institute of Technology; January, 1969. Adviser: Professor Andre G. Vacroux.

Prefaced by a literature survey of earlier applications of modern control theory and presentation of pertinent kinetics equations, the dissertation describes the sequential analytical investigation of a digital computer control system to implement nuclear reactor control and estimation functions.

First, nonlinear plant and measurement equations are derived for a deterministic one-group prompt-jump point model, using rate of reactivity change as control input. Next, state-space concepts are introduced, resultant equations are expressed in vector-matrix notation, linearized by a first-order Taylor series expansion, and solved for a discrete-time input.

Dynamic programming yields an optimal stationary feedback control law which minimizes a quadratic performance index for a discrete-time system. An index consisting of the sum squares of the neutron density derivations is defined and augmented to include terms in reactivity and control input. With the aid of an iterative digital computer program the stationary feedback matrix is calculated for selected values of weighting coefficients. Corresponding transient behavioral plots of the nonlinear system show that for the performance index as defined, the

neutron density deviation is decreased to zero in one sample interval after a step disturbance in reactivity.

In order to satisfy the optimal control law requirement that all state variables be available, a nonlinear estimator is used to generate estimates of nonmeasurable system state variables. Estimator equations, based on a set of finite-difference equations, are derived by minimizing a performance index consisting of the sum squares of errors in the previous estimate and in the current measurement. The resulting nonlinear equations are solved iteratively on a digital computer. Since the system is described by differential equations, integration is used to obtain the numerical values required by the estimator during the iteration sequence.

Finally, the cascade combination of an optimal estimator and optimal controller yields a control system whose performance is unequal to a system without an estimator. Estimates generated for the nonlinear system necessitate a large control input at the first sampling following a reactivity disturbance. Inclusion of a computation time delay results in further degraded performance. If an integrator is incorporated into the nonlinear estimator, the integration step size must be reduced when a control input is present. Since the computer programs used to solve the estimator equations and to compute the control input are not compiled for minimum time execution, no conclusion can be made with regard to real-time control capability.

The dissertation includes a comprehensive literature survey of earlier applications of modern control theory to nuclear reactors, a detailed review of pertinent reactor kinetics equations, and a wealth of selected nuclear and control engineering bibliographies.

CHAPTER 1

INTRODUCTION

1.1 Growth of nuclear power plants

Achievement of the first self-sustaining nuclear fission chain reaction in 1942 was recognized by Enrico Fermi and his colleagues as the initial objective toward creation of a destructive weapon. However, each scientist also recognized the constructive potential of controlling and converting the heat of fission into useful mechanical and electrical energy. In fact, one of the earliest concepts of converting nuclear energy into useful electrical energy — the Daniels Experimental Power Pile at Oak Ridge National Laboratory — was based on studies initiated in 1944 by Dr. Farrington Daniels, a member of this historic group. Unfortunately, national security prevailed and the application of controlled nuclear power was directed toward military logistics.

In 1947, Congress authorized the development of a nuclear reactor for submarine propulsion. Work initiated at Argonne National Laboratory near Lemont, Illinois, led to the construction and operation, on March 30, 1953, of the first nuclear propulsion system in a section of a submarine hull at the National Reactor Testing Station in Idaho. This land-based installation was the forerunner of the pressurized water system used in the submarine Nautilus, which was launched the following year. This launching represented the first milestone of the Naval Reactors Program which has since revolutionized naval strategy.

New reactor concepts for municipal power systems also were pioneered by Argonne scientists and engineers through the design, development, construction, and operation of simplified experiments or small-scale prototype systems at the Argonne test site in Idaho. Such was the case in 1951, when Experimental Breeder Reactor-I became the first nuclear reactor to generate electricity (170 kilowatts), thereby demonstrating the technical feasibility of: using unmoderated reactors for generation of useful power, employing sodium and sodium-potassium alloy as coolants, and breeding plutonium fuel. This experiment led the way to subsequent construction and operation, in 1963, of: EBR-II, a prototype fast power breeder central station plant; and the Enrico Fermi Atomic Power Plant, the world's first large fast breeder nuclear power plant.

In 1953, a series of Boiling Reactor Experiments (BORAX-I, -II, -III) were started at the Idaho test site. These experiments ultimately demonstrated the inherent power stability of the boiling water reactor concept. On July 17, 1955, the town of Arco, Idaho, was temporarily serviced with electricity generated by the BORAX-III power plant.

The technology gained from the BORAX experiments was applied in the construction of the Experimental Boiling Water Reactor (EBWR) at Argonne. On December 29, 1956, EBWR achieved its rated electrical output of 5,000 kilowatts, and thus became the first of a series of prototype central station power reactors to go into operation in the USAEC Civilian Power Reactor Development Program.

Two years later (May, 1958), the Shippingport Atomic Power Station in Pittsburgh, Pa., was dedicated as the first large-scale, nuclear power-generating plant (60,000 electrical kilowatts) in the

United States. Built by Westinghouse Electric Corporation as part of the same Civilian Power Reactor Development Program, the Shippingport plant design is based on the pressurized, light-water reactor concept.

Since 1958, the growth of nuclear powered central station plants in the United States has exceeded early predictions. This growth has been achieved by making nuclear plants economically competitive with conventional fossil-fueled plants. The most recent survey [1]* lists 13 operable, 31 being built, and 40 planned. Of these plants, 81 are based on the boiling and pressurized light-water reactor concepts.

As a consequence of the ever-increasing demand for uranium to fuel the light-water-cooled reactor power plants, the U.S. Atomic Energy Commission (USAEC) has given the highest priority to development of liquid-metal-cooled fast breeder reactors. In August, 1968, a Liquid-Metal Fast Breeder Reactor (LMFBR) program plan was issued. The overall objective is to achieve, through research and development, the technology required to design, construct, and safely, reliably, and economically operate fast breeder reactors for use in central station nuclear power plants. Volume 4 of that plan specifies the instrumentation and control developments essential to reliable and safe operation of an LMFBR plant [2].

1.2 Outline of dissertation

The research described in this dissertation was undertaken with the objective of applying modern control theory to the analysis and design of an optimal control system for a liquid metal fast breeder reactor. The fundamental problems of finding the optimal regulator control law and of estimating the states of the nonlinear deterministic system model

*Numbers in brackets pertain to references cited on pages 164 to 173.

have been solved. A natural consequence of applying dynamic programming to obtain the feedback regulator solution and iteration to the estimation problem is the requirement that a digital computer be used to implement the control and estimation functions.

Chapter 2 is devoted to a review of earlier applications of optimal control theory to nuclear reactor control problems. Since it was not feasible to discuss the specific applications in detail, appropriate references are cited. In addition, extensive selected bibliographies of nuclear and control engineering literature have been compiled for those who wish to specialize in this area.

Chapter 3 contains the equations which describe the reactor system. A one-group delayed neutron model is used as an approximation to the six-group system. A further simplification of the system equations is achieved by using a prompt-jump approximation.

In Chapter 4, the system differential equations are defined in terms of state variables and matrices. Nonlinear system equations are linearized using nominal values and the resulting set of equations is solved with discrete-time inputs.

Chapter 5 treats the solution of the closed loop regulator problem by applying dynamic programming to obtain the minimum of a specified performance index and the resulting transient response is discussed.

The closed loop solutions of Chapter 5 idealistically assume that all state variables are measurable; therefore, the solution of a deterministic estimator is derived in Chapter 6. Chapter 7 considers the combined problem of estimation and control.

Finally, the work is summarized, along with conclusions and recommendations for future research, in Chapter 8.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

From 1942 to 1960, analysis and design of control systems for nuclear reactors was based on classical methods.

Modern reactor control theory, which is concerned with optimal processes, emerged from Wiener's [6] theory in 1942, Bellman's [7] dynamic programming techniques in 1954, and Pontryagin's [8] Maximum Principle in 1956. Although several papers on off-line optimization of nuclear fuel management and xenon shutdown programs were published, Kallay [3], in 1960, was the first to relate modern control theory to nuclear reactors.

Early application of digital computer techniques to power reactors was limited primarily to data handling and on-line computations. In 1962, an issue of Nucleonics [4] was devoted to a special report on on-line computers for power reactors. At the 1964 Geneva Conference, Schultz and Legler [5] presented a status report on the application of digital computer techniques to reactor operation. Today, computer control systems are installed on several nuclear reactors, but these installations are on critical facilities or limited only to process control on power reactors. Literature describing these systems are listed in the general nuclear bibliography.

2.2 Previous investigations

Kallay [3] suggested four applications of dynamic programming techniques to nuclear reactors: optimization of poison distribution, optimization of over-all plant efficiency with respect to component cost, design of optimal control programs, and determination of flow distribution through a heat exchanger. Under control applications, Kallay outlined the optimal solution to a minimum energy start-up problem.

Foureau [9] used Pontryagin's maximum principle, a single group of delayed neutrons, and a constraint on the rate of change of reactivity, to determine the switching boundaries for a reactor start-up program.

Shen and Haag [10, 11, 12, 14] and Haag [13] used Pontryagin's maximum principle to solve an optimum start-up problem using a one-group delayed neutron model and a prompt-jump approximation. In the resulting control scheme, the switching conditions on the input were determined by nonlinear functions of time.

Mulcahey [15, 16] analyzed the time optimal control of nuclear reactors with velocity-limited control devices. His model consisted of a fast reactor with one group of delayed neutrons and a reactivity feedback, which was a function of the power level. The prompt-jump approximation was employed, and the resulting set of equations was solved analytically. System behavior was studied with analog and digital computers. He concluded that a power-level-based switching controller should be adopted.

Rosztoczy [17, 18] used the maximum principle and analyzed three optimization problems: a shutdown program for minimum xenon buildup, flux state changes in nuclear reactors, and minimum fuel loading. The

model consisted of a single group of delayed neutrons and a reactivity feedback proportional to the power level. An integral performance index equal to reactivity squared was minimized by solving the resulting two-point boundary value problem on an analog computer. A suboptimal minimum-time solution was investigated by decreasing the time to execute a change in power level. Power level changes with minimum control energy were investigated by assuming a performance index equal to the integral of the reactivity rate squared. The solutions presented were open loop, and the control input was generated as a function of the adjoint variables.

Ruiz [19] used Pontryagin's maximum principle to minimize an integral performance index consisting of the sum of power deviation squared and square of the product of reactivity and power. One group of delayed neutrons was assumed. A closed loop control law was derived which required pre-programmed time variable coefficients.

Ash [20] used dynamic programming to derive a functional equation which would cause a boiling reactor to be driven back to its equilibrium condition in minimum time by continuously moving control rods.

Hermsen [21] used Wiener's theory and a linearized model of the reactor to design a closed loop control system based on minimization of an integral squared error index. Also, Z transform theory was used to design a control system which would be suitable for computer control. Pontryagin's maximum principle was applied to a system consisting of six groups of delayed neutrons and a model based on Newton's law of cooling. A set of $2(m + 7)$ equations resulted, where m was the number of temperature nodes. Dimensionality of the problem was reduced by going to a one-group linearized model, and a closed-loop control law was

derived. The maximum principle also was used to solve the minimum-time problem with and without a constraint on the reactivity rate. In view of the difficulties encountered in obtaining solutions, it was suggested that dynamic programming be applied to the problem in future research.

Kliger [22, 42] used Holder's inequality to solve the minimum-time control problem subject to a constraint consisting of the product of reactivity and flux. One group of delayed neutrons was assumed. He derived a closed loop switching function, and proposed that a state estimator be used to generate the non-measurable state variables.

Mohler [22, 24, 25] used the maximum principle to analyze the minimum-time control of neutron density subject to a magnitude constraint on reactivity. A bang-bang control law was derived. In order to maintain constant power level, an additional input was required, after the last switching, to offset the effect of delayed neutrons. For the case of a six-group delayed neutron model, a feedback reactivity proportional to the sum of the rate of change of precursors was required to hold power level constant. A dither control was proposed as an alternate solution.

Weaver et al. [26] investigated: suboptimal closed-loop control employing the second method of Lyapunov, nonlinear stability of coupled core reactors described by a set of differential-difference equations, synthesis of optimal closed-loop control of nuclear reactor systems, and limits of validity for some approximations in reactor dynamics.

Secker and Weaver [27, 28] investigated optimal closed-loop control using a set of equations linearized around a nominal trajectory, and a quadratic performance index. Application of Pontryagin's maximum principle led to a matrix Riccati equation. The optimal filter for state-variable estimation was derived using Kalman's method for

differential systems, and a matrix Riccati equation was solved for the optimal gain. The resulting closed-loop control system required storage of the preprogrammed control variable and nominal state trajectory.

Melsa [29, 30] extended the work reported previously by Weaver et al. [26]. Suboptimal control with a singular control matrix was investigated and applied to the control of a nuclear rocket.

Kliger [31] defined a control variable which was equal to the product of neutron flux density and reactivity and made the neutron kinetics equations linear. Reactivity was recovered as a true input control quantity by dividing the control variable by the measured flux. He applied the maximum principle to the problem using an integral performance index, and obtained the optimal control function in terms of the state and adjoint variables. Using back substitution, he then solved for the control function in terms of the state variables. An estimator was designed to generate the delayed neutron states from neutron flux measurements.

Duncombe [32, 33, 34] used the same linearizing approximation as Kliger to investigate on-line optimization of nuclear reactor load control in the presence of nonlinearities. To carry through this simplification, the performance index included a term of reactivity times flux squared. Based on this approximation, the results obtained by Duncombe must be judged accordingly. The optimal closed loop solution was obtained by using the maximum principle and deriving a matrix Riccati equation. The solution of the matrix Riccati equation varied with the varying load demand. To apply the correct feedback at each instant, it was necessary to calculate the parameters of the feedback network in effectively zero time. An analog computer was used to solve

the matrix Riccati equation in 0.1 real time and to simulate the reactor plant. All of the state variables were obtained from the simulation. In his conclusions, Duncombe pointed out that in an actual application, the reactor plant simulation would be replaced by the reactor itself, however, he did not state that a state estimator would be necessary to generate non-measurable variables.

Monta and Lennox [35] investigated time-optimal digital computer control for the NRU reactor by applying the method of Desoer and Wing [36].

Kliger [37] extended his work [31] to analysis of an optimal control system for nuclear reactors with a generalized temperature feedback. The problem was subdivided such that a specific controller yielded the coolant flow and neutron density to minimize a performance index, and a universal controller forced the reactor neutron density to follow the desired neutron density. The maximum principle was applied, and the resulting set of equations was solved to obtain the optimal control law. The control law required all state variables, so an estimator was designed to generate delayed neutron estimates from neutron flux measurements.

Sokolova [38] analyzed the problem of determining an optimum control law for a nuclear power plant. A set of 29 differential equations, bilinear in the state variables and in the state and two control variables, was used to describe the plant, which consisted of a reactor, a regenerator, a cooler, and a turbocompressor. A quadratic performance index was used, and dynamic programming was applied. Two control equations were derived: one linear in the state variables and the other nonlinear. Lyapunov's method was applied to guarantee stability of the

control system. Implementation of the control scheme required that all state variables be measurable.

Weaver et al. [39] investigated: optimal feedback control of nuclear reactor systems, modeling with Lyapunov functions, and linear system design using state variable feedback. The optimal control investigation used the linearizing substitution of Kliger [31]. A quadratic error index and prompt reactor model were used and a time-varying gain was obtained for the optimal feedback control by means of Bellman's equation. The analysis was repeated on reactor models using prompt nonlinear, linear delayed, and nonlinear delayed neutrons, with and without feedback. The developed methods were then used to analyze the start-up of a nuclear rocket.

Higgins [40] and Higgins and Schultz [41] investigated the stability of certain nonlinear time-varying systems of automatic control. They used the second method of Lyapunov, the Popov frequency criterion, and the matrix inequality method. As an example, the stability theory was applied to the simplified nuclear rocket propulsion system considered by Mohler (1962).

Monta [43, 44, 45] investigated the time-optimal control of nuclear reactors. One group of delayed neutrons and a prompt jump approximation were assumed. The maximum principle was used to derive the switching trajectories in state space, with and without constraints. The discrete version of the maximum principle was used to analyze a system with a pulse-width-modulated-reactivity input. An experiment was performed on the Toshiba Training and Research Reactor using a digital control computer. Computing time delay, control rod motor time constant,

one-group approximation, and reactivity estimates had to be taken into account for practical reasons.

Humphries [46, 47] used a parameter adjustment model to investigate adaptive control of a nuclear rocket engine. The proportional control gain for the control poison was the parameter adaptively adjusted and the maximum core surface temperature was the variable adaptively controlled. The performance index consisted of the integral squared response error, which was formed by comparing the system output with that of the reference model. To evaluate the performance index, the nuclear rocket engine equations were linearized, the prompt neutron lifetime was set equal to zero, and the effects of delayed neutrons were neglected. Parseval's theorem was used to evaluate the performance index as a function of gain. It was shown that propellant savings of up to 20,000 pounds per transition from idle to full power are possible with adaptive control.

Saluja [48], and Saluja, Sage, and Uhrig [49] analyzed open and closed-loop control of nuclear systems. Three performance indices were considered: integral of reactivity squared, integral of reactivity squared and neutron density deviation squared, and the previous index with reactivity set equal to a proportional flux integral function of neutron density error. The maximum principle was applied, and quasi-linearization was used to solve the resulting two-point boundary value problem. Convergence was obtained in no more than four iterations for all problems. The suboptimal closed-loop control law yielded poorer performance than the open-loop control law. It was suggested that an adaptive-type control be considered to improve performance.

Ellis [50], and Sage and Ellis [51] presented a sequential suboptimal adaptive control philosophy which encompassed both identification and control. A general nonlinear differential system was modeled by a linear time varying system of assumed form. The system was assumed stationary over subintervals of time. This allowed a controller to generate a sequential control law which minimized an integral of time weighted quadratic form of error and control effort. The method was used to generate an optimum closed-loop control for the start-up dynamics of a nuclear reactor system.

Masters [52], and Sage and Masters [52] derived a sequential method for on-line estimation of the state variables and parameters of discrete, nonlinear, dynamic systems. The discrete version of the maximum principle was employed to obtain the canonic equations of the least-squares optimal estimator. Also, a discretized invariant imbedding technique was applied to solve the resulting two-point boundary value problem. A system of sequential equations was then obtained by application of variational methods to the optimal trajectory. The estimation procedure provided the best least-squares estimate of the state vector, given noisy measurements at discrete intervals of time. The method was applied to a nuclear reactor, with a single group of delayed neutrons, and the system state and one parameter were estimated.

Ogawa, Kaji, and Ozawa [54] analyzed the time-optimal control of nuclear reactors with two kinds of internal feedback: a prompt feedback generated by variations of fuel temperature and coolant density, and a delayed feedback governed by variations of moderator temperature. System stability was examined by investigating the behavior of the

linearized system near an equilibrium point. The maximum principle was applied to the quasilinear system to obtain the optimum control law.

Rasetti and Vallauri [55] discussed the maximum principle and dynamic programming. A nuclear propulsion plant for a commercial ship with four steam generators and one pressurizer was analyzed for time-optimal control using the maximum principle. The canonical equations were compared to the results obtained by applying Bellman's equation.

Tataru, Bajenescu, and Ghetaru [56] considered the closed-loop regulator problem of a nuclear reactor. The small signal transfer function of a reactor was used. A scheme was derived to keep the loop gain constant by using a perturbing signal and a computing device to offset gain changes caused by power level changes.

Partain [57], and Partain and Bailey [58, 59] studied the application of Z transforms to linearized kinetics equations. Digital simulation was used to investigate system behavior.

Herring [60], Herring et al. [61], Weaver [62] and Weaver and Vanasse [68] developed a method for designing control systems by using state variable feedback. This method was applied to a two-temperature-region reactor and to a coupled-core reactor. Linearized transfer functions were used for the reactor systems. A method also was outlined for generating non-measurable state variables by placing frequency dependent elements in the feedback path.

Miyazaki [63] applied Wiener's theory [6] of least-squares optimization with quadratic constraint to the design of reactor control systems. The deterministic case was investigated by taking the integral square error for the criterion function and the integral square of reactivity rate for the control function. The stochastic case was

studied by substituting the mean-square error and mean-square reactivity rate, respectively. Transfer functions for various step sizes and ramp inputs were derived.

Habegger [64], and Habegger, Bailey, and Kadavanich [65] applied quasilinearization and Kalman filter techniques to estimate nuclear parameters in the EBWR, PUR-I, and EBR-II reactors.

Melsa et al. [66] investigated: system identification using a random search method, data reconstruction using non-resetting integrators, and sub-optimal closed-loop control using invariant imbedding.

Mohler [67] analyzed the fuel-optimal control of a nuclear propulsion system by means of the maximum principle, Lagrange multipliers and computers. Practical problems were shown to be complicated by state constraints and high dimensionality. A minimum-time, prompt-neutron control process with reactivity rate and amplitude constraint was analyzed.

Mohler and Price [69, 70, 102] investigated application of linear programming procedures to optimal control of nuclear rocket reactors which had inequality magnitude constraints imposed on the control and state. Nonlinear equations were transformed into a form suitable for linear programming by using a first-order Taylor series expansion.

Marciniak [71, 101] studied the time-optimal digital control of zero power nuclear reactors. Sampled-data control system theory, including Z-transforms and discrete state variables, was used to design a control system which would: increase power level while maintaining a minimum period, and reach demand power level with little, or no, overshoot. Of the various data-holds investigated, the zero-order hold

was the most stable. A time optimal study was made of a one-group delayed neutron reactor using the maximum principle, and the switching equation was derived. This switching equation and the zero-order hold were used to derive a control program, which was applied to noise-free reactor models simulated on a digital computer. A modified version of the control program was used on the Argonne Thermal Source Reactor.

CHAPTER 3

REACTOR DYNAMICS

3.1 Introduction

The derivation of the nuclear reactor kinetics equations, starting from neutron physics fundamentals, is well documented. These include treatments of the subject by: Glasstone and Edlund [72, Weinberg and Wigner [73], Meghreblian and Holmes [74], Isbin [75], or Ash [76], and a handbook presentation by Radkowsky [77]. An excellent treatment on general reactor dynamics is given by Gyftopolous [78], and the specific subject of fast reactor kinetics is treated by McCarthy and Okrent [79]. A discussion of the general subject of reactor dynamics and control is given by: Ash [76], Harrer [80], Keepin [81], Schultz [82], and Weaver [83, 84].

3.2 Six-group delayed neutron model

The point-model kinetics equations for a nuclear reactor are:

$$\frac{dn(t)}{dt} = \frac{\delta k(t) - \beta}{\ell} n(t) + \sum_i \lambda_i c_i(t) \tag{3.1}$$

and

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\ell} n(t) - \lambda_i c_i(t) \qquad i = 1,\ldots,6 \tag{3.2}$$

where

$n(t)$ = neutron density

$\delta k(t)$ = reactivity

β = total delayed neutron fraction

ℓ = neutron lifetime

λ_i = decay constant of the ith neutron precursor

$c_i(t)$ = concentration of delayed neutrons of group i

β_i = delayed neutron fraction of group i

Reactor power level is proportional to neutron density. At low power levels, reactivity is not a function of the neutron density; therefore Eqs. (3.1) and (3.2) are commonly referred to as the zero power kinetics equations.

In Eq. (3.1) reactivity is a function of time, and for this condition, Eqs. (3.1) and (3.2) are linear with time varying coefficients. At high power levels, reactivity is a function of the neutron density, and the equations become nonlinear.

The values of λ_i and β_i for U-235 fueled fast reactors [85, p. 18] are listed in Table 3.1.

TABLE 3.1

DELAYED NEUTRON YIELD FROM FAST FISSION IN U-235

λ_i	β_i	a_i
0.0127	0.000247	0.038
0.0317	0.00138	0.213
0.115	0.00122	0.188
0.311	0.00265	0.407
1.40	0.000832	0.128
3.87	0.000169	0.026

The relative abundance is given by $a_i = \beta_i/\beta$. The total delayed neutron fraction is obtained from $\beta = \Sigma\beta_i$, and for the

values of β_i in Table 3.1, $\beta = 0.0065$. Typically, $\ell = 10^{-7}$ sec for a fast reactor.

If the following variables are defined

$$\alpha = \beta/\ell \tag{3.3}$$

$$\alpha_i = \beta_i/\ell \tag{3.4}$$

$$\rho(t) = \delta k(t)/\beta \tag{3.5}$$

and substituted into Eqs. (3.1) and (3.2), then

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \sum_i \lambda_i c_i(t) \tag{3.6}$$

$$\dot{c}_i(t) = \alpha_i n(t) - \lambda_i c_i(t) \qquad i = 1,\ldots,6 \tag{3.7}$$

where the dot notation designates the derivative with respect to time, and ρ is reactivity in dollars. Typically, $|\rho|<1$.

At equilibrium, the time derivatives are equal to zero, which on solving Eq. (3.7) gives

$$c_i(0) = \alpha_i n(0)/\lambda_i \tag{3.8}$$

The delayed neutron concentration can be normalized by defining

$$z_i(t) = (\lambda_i/\alpha)\, c_i(t) \tag{3.9}$$

Substitution of Eq. (3.9) into Eqs. (3.6) and (3.7) results in a set of normalized equations

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \alpha \sum_i z_i(t) \tag{3.10}$$

$$\dot{z}_i(t) = \lambda_i [a_i n(t) - z_i(t)] \qquad i = 1,\ldots,6 \tag{3.11}$$

where the equilibrium solution requires that $z_i(0) = a_i n(0)$ and $\sum_i z_i(0) = n(0)$ because $\sum_i a_i = 1$.

3.3 Transient response of six-group model

For a step input of reactivity, the kinetics equations can be solved by application of the Laplace transform. Under the conditions of a step input $\rho(t) = \rho$, a constant. This constant value of reactivity is substituted into the equation before transformation. The initial conditions of $n(0)$ and $z_i(0)$ are the values of $n(t)$ and $z_i(t)$ which exist just prior to the step addition of reactivity.

With ρ set equal to a constant, taking the Laplace transform of Eqs. (3.10) and (3.11) results in

$$sN(s) - n(0) = \alpha\rho N(s) - \alpha N(s) + \alpha \sum_i Z_i(s) \tag{3.12}$$

$$sZ_i(s) - z_i(0) = \lambda_i[a_i N(s) - Z_i(s)] \qquad i = 1,\ldots,6 \tag{3.13}$$

Equation (3.13) is solved for $Z_i(s)$ to give

$$Z_i(s) = \frac{a_i\lambda_i}{s + \lambda_i} N(s) + \frac{z_i(0)}{s + \lambda_i} \qquad i = 1,\ldots,6 \tag{3.14}$$

Equation (3.14) is then substituted into Eq. (3.12) to obtain an equation for $N(s)$. Thus

$$N(s) = \frac{n(0) + \alpha \sum_{i=1}^{6} \frac{z_i(0)}{s + \lambda_i}}{s + \alpha(1 - \rho) - \alpha \sum_{i=1}^{6} \frac{\lambda_i a_i}{s + \lambda_i}} \tag{3.15}$$

Remembering that $\sum_i a_i = 1$, the denominator of Eq. (3.15) can be rearranged to yield:

$$N(s) = \frac{n(0) + \alpha \sum_{i=1}^{6} \frac{z_i(0)}{s + \lambda_i}}{s - \alpha\rho + \alpha \sum_{i=1}^{6} \frac{a_i s}{s + \lambda_i}} \tag{3.16}$$

Equation (3.16) is valid for any arbitrary initial conditions of $n(0)$ and $z_i(0)$. If the system is at equilibrium before the reactivity

addition, then

$$z_i(0) = a_i n(0) \tag{3.17}$$

and substitution of Eq. (3.17) into Eq. (3.16) results in an expression of N(s) as a function of the initial neutron density. Thus

$$N(s) = \frac{1 + \alpha \sum_{i=1}^{6} \frac{a_i}{s + \lambda_i}}{s - \alpha\rho + \alpha \sum_{i=1}^{6} \frac{a_i s}{s + \lambda_i}} n(0) \tag{3.18}$$

In order to find the inverse Laplace transform of Eq. (3.18), the roots of the denominator must be known. If the numerator and denominator of Eq. (2.18) are multiplied by the factors $s + \lambda_i$, a seventh-order polynomial in s is obtained for the denominator, with coefficients consisting of complicated combinations of products and sums of the λ_i [82, pp. 110-111]. This polynomial is then factored for the roots.

An alternate method is to apply iteration to the denominator of Eq. (3.18) by means of the Newton-Raphson algorithm [86, p. 78] as follows:

$$s_{n+1} = s_n - \frac{F(s_n)}{F'(s_n)} \tag{3.19}$$

which converges quadratically to yield the solution of $F(s_{n+1}) = 0$ with

$$F(s) = s - \alpha\rho + \alpha \sum_i \frac{a_i s}{s + \lambda_i} \tag{3.20}$$

$$F'(s) = 1 + \alpha \sum_i \frac{a_i \lambda_i}{(s + \lambda_i)^2} \tag{3.21}$$

where F(s) is the denominator of Eq. (3.18) and F'(s) is the derivative of F(s) with respect to s.

Substitution of Eqs. (3.20) and (3.21) into Eq. (3.19) results in

$$s_{n+1} = s_n - \frac{s_n - \alpha\rho + \alpha \sum_i \frac{a_i s_n}{s_n + \lambda_i}}{1 + \alpha \sum_i \frac{a_i \lambda_i}{(s_n + \lambda_i)^2}} \tag{3.22}$$

which can be rearranged as follows:

$$s_{n+1} = \frac{\rho - \sum_i \frac{a_i s_n^2}{(s_n + \lambda_i)^2}}{\frac{1}{\alpha} + \sum_i \frac{a_i \lambda_i}{(s_n + \lambda_i)^2}} \tag{3.23}$$

In order for Eq. (3.23) to converge, suitable initial values must be chosen for the various roots. For positive ρ, one root is positive and all others are negative and range between the λ_i values [76, p. 32]. For ρ negative, all seven roots are negative. The most negative root is approximately equal to $\alpha(\rho - 1)$.

Equation (3.18) can be expressed as a partial fraction expansion. That is,

$$N(s) = \sum_{i=1}^{7} \frac{B_i}{s - s_i} n(0) \tag{3.24}$$

Since the poles of Eq. (3.18) are simple, the coefficients B_i of Eq. (3.24) can be obtained from:

$$B_i = \left. \frac{1 + \alpha \sum_i \frac{a_i}{s + \lambda_i}}{F'(s)} n(0) \right|_{s = s_i} \tag{3.25}$$

where $F'(s)$ is given by Eq. (3.21).

The Roots of Prompt Jump Equation computer program which finds the s_i and calculates the corresponding B_i is listed in Appendix G. Table 3.2 lists the s_i and B_i for a step input of $\rho = 0.1$.

TABLE 3.2

ROOTS OF KINETICS EQUATIONS AND TRANSIENT RESPONSE COEFFICIENTS FOR $\rho = 0.1$

s_i	B_i
.01046741	1.2924847
-.01438199	-0.03533592
-.06525568	-0.08955314
-.19093692	-0.04046886
-1.2253240	-0.01346368
-3.7713468	-0.00255375
-58,500.482	-0.11110930

The solution for the neutron density as a function of time, obtained by taking the inverse transform of Eq. (3.24), is

$$n(t) = \sum_{i=1}^{7} B_i e^{s_i t} \tag{3.26}$$

The time constant corresponding to the most negative root in Table 3.2 is 17 μsec. If Eq. (3.26) is evaluated at t = 0.001 sec, using the values in Table 3.2, n(0) = 1.0, and the Reactor Response to Step Delta K computer program listed in Appendix G, then n(0.001) = 1.111. The flux has jumped 11.1% in 1 msec, and remains at this level until the terms in Eq. (3.26) with longer time constants began to exert their influence.

3.4 Prompt-jump approximation

In the analysis which follows, detailed reactor transient behavior at times less than 1 msec will not be of interest. Transient behavior in this case can be adequately described by employing the prompt-jump approximation. Setting $\dot{n}(t) = 0$ in Eq. (3.10) results in

$$0 = \alpha\rho(t)n(t) - \alpha n(t) + \alpha \sum_{i=1}^{6} z_i(t) \tag{3.27}$$

which is then solved for n(t):

$$n(t) = \frac{\sum_{i=1}^{6} z_i(t)}{1 - \rho(t)} \tag{3.28}$$

The neutron density is eliminated from Eq. (3.11) by substituting Eq. (3.28) for n(t) to obtain

$$\dot{z}_i(t) = \frac{\lambda_i a_i \sum_{i=1}^{6} z_i(t)}{1 - \rho(t)} - \lambda_i z_i(t) \tag{3.29}$$

Reactor response to a step input can be determined by means of Eqs. (3.28) and (3.29). For the case of equilibrium conditions prior to the step, $\rho = 0$ and $\sum z_i(0-) = n(0-)$. Immediately after the step

$$n(0+) = \frac{1}{1 - \rho} n(0-) \tag{3.30}$$

and n(t) has increased by the factor $1/(1 - \rho)$. If $\rho = 0.1$,

$$\frac{1}{1 - \rho} = 1.111 \tag{3.31}$$

which is the same as the transient response calculated previously for $t = 0.001$ sec and $n(0) = 1.0$.

3.5 One-group delayed neutron model

A further reduction in system dimensionality can be achieved by considering a single group of delayed neutrons. With this assumption, Eqs. (3.10) and (3.11) become

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \alpha z(t) \tag{3.32}$$

$$\dot{z}(t) = \lambda[n(t) - z(t)] \tag{3.33}$$

The single-group decay constant λ must be suitably chosen if the one-group approximation is to provide useable results. In previous applications of the approximation, λ has been selected on the basis of best asymptotic behavior as $t \to \infty$. This method of selection is not the best for studying transient behavior at times of the order of one second; therefore an alternate method based on a matching of the transient response is proposed.

3.6 Transient response of one-group model

The transient response of the one-group model to a step input of reactivity can be determined by taking the Laplace transform of Eqs. (3.32) and (3.33) or equivalently modifying the six-group result of Eq. (3.16) to give

$$N(s) = \frac{n(0) + \dfrac{\alpha z(0)}{s + \lambda}}{s - \alpha\rho + \dfrac{\alpha s}{s + \lambda}} \tag{3.34}$$

which alternately can be written

$$N(s) = \frac{(s + \lambda)n(0) + \alpha z(0)}{s^2 + (\lambda + \alpha - \alpha\rho)s - \alpha\rho\lambda} \tag{3.35}$$

Given the numerical values of λ, α, and ρ, the roots of Eq. (3.35) may be calculated directly. These roots may be approximated by using

the quadratic formula and the product relationship of the roots to obtain

$$s_1 \approx \lambda\rho/(1 - \rho) \tag{3.36}$$

$$s_2 \approx -\alpha(1 - \rho) - \lambda/(1 - \rho) \tag{3.37}$$

assuming that $\lambda << \alpha$.

The partial fraction expansion and inverse transformation of Eq. (3.35), using the roots given by Eqs. (3.36) and (3.37), results in

$$n(t) = \frac{\lambda e^{s_1 t} + [\alpha(1 - \rho)^2 + \lambda\rho]e^{s_2 t}}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)} n(0) + \frac{\alpha(1 - \rho)[e^{s_1 t} - e^{s_2 t}]}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)} z(0) \tag{3.38}$$

3.7 Transient response of one-group prompt-jump model

The prompt-jump approximation can be applied to Eq. (3.32) by setting $\dot{n}(t) = 0$ and solving for $n(t)$. Then

$$n(t) = \frac{z(t)}{1 - \rho(t)} \tag{3.39}$$

This solution for $n(t)$ is substituted into Eq. (3.33) to obtain an equation in $z(t)$ and $\rho(t)$. Thus

$$\dot{z}(t) = \frac{\lambda\rho(t)z(t)}{1 - \rho(t)} \tag{3.40}$$

The solution of Eq. (3.40) is

$$z(t) = z(0) \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)} dt \tag{3.41}$$

and the flux density solution is obtained by substituting Eq. (3.41) into Eq. (3.39) to obtain

$$n(t) = \frac{z(0)}{1 - \rho(t)} \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)}\, dt \qquad (3.42)$$

If $\rho = 0$ for $t<0$, then $z(0) = n(0)$, and Eq. (3.42) becomes

$$n(t) = \frac{n(0)}{1 - \rho(t)} \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)}\, dt \qquad (3.43)$$

If reactivity is constant, then $\rho(t) = \rho$, and Eq. (3.43) becomes

$$n(t) = \frac{n(0)}{1 - \rho} \exp [\lambda\rho t/(1 - \rho)] \qquad (3.44)$$

The same result is obtained from Eq. (3.38) for $t>0.001$ sec because the contribution from the second exponential term is then negligible.

3.8 Selection of one-group decay constant

In later analyses, reactor transient behavior will be examined in response to input signals occurring at one second intervals. It is therefore desirable to select a λ which will provide the best approximate transient response at the end of one second. For the case of $\rho = 0.1$, $n(0) = 1.0$, and $t = 1$ sec, Eq. (3.43) is set equal to Eq. (3.26) using the values in Table 3.2. This results in $\lambda = 0.312$. This value of λ will be used in subsequent calculations which utilize the single-group model. Note that, within accuracy limits, this particular value of λ coincides with one of the intermediate values of λ listed in Table 3.1.

3.9 Reactivity input

Reactivity changes in an actual system are effected by a control rod mechanism. Figure 3.1 shows a block diagram of a reactivity input system.

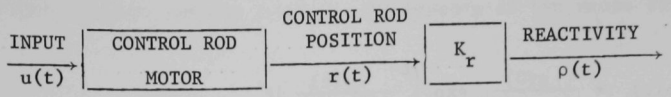

Fig. 3.1 Reactivity input system

The gain K_r has been included in Fig. 3.1 to account for the control rod calibration in terms of units of reactivity for units of position.

The control rod motor transfer function is given by

$$\frac{R(s)}{U(s)} = \frac{K_m}{s(1 + sT_m)} \tag{3.45}$$

which can be expressed as a differential equation as follows:

$$\dot{r}(t) + T_m\ddot{r}(t) = K_m u(t) \tag{3.46}$$

If it is assumed that the motor time constant is negligible, then Eq. (3.46) reduces to

$$\dot{r}(t) = K_m u(t) \tag{3.47}$$

Reactivity is related to control rod position by

$$\rho(t) = K_r r(t) \tag{3.48}$$

which upon substitution into Eq. (3.47) yields

$$\dot{\rho}(t) = K_m K_r u(t) \tag{3.49}$$

If $K_m K_r$ is set equal to one, then the units of u(t) are given directly in dollars per second, and Eq. (3.49) becomes

$$\dot{\rho}(t) = u(t) \tag{3.50}$$

Equation (3.50) shall be used in subsequent analysis to express the functional dependence of reactivity on an input.

CHAPTER 4

STATE SPACE REPRESENTATION OF REACTOR DYNAMICS

4.1 Introduction

The classical methods of control system analysis and design are based on input-output relationships of systems generally represented by one *n*th order differential equation. Modern control theory utilizes the concepts of state space and state variables, and an *n*th order system is represented by a set of n first-order differential equations.

The selection of a set of state variables to represent a system described by one *n*th order differential equation is not unique. In the case of reactor kinetics, formulation of system equations from physical considerations has led to a natural selection of state variables, and the system is initially described by n first-order differential equations.

It is convenient to first apply the concept of state space to a reactor with one group of delayed neutrons and then extend it to a reactor with six groups. For the one-group reactor, the neutron density n(t) and delayed neutron precursor density c(t) are the two variables which uniquely describe the state of the reactor at any time t. The state space for the reactor is two dimensional, a plane, and its coordinates are n(t) and c(t). The two coordinates are specified by a pair of ordered numbers, a vector. The state of the reactor at any time t can be associated with a point in a plane. Given $n(t_0)$ and $c(t_0)$,

which determine the reactor state at any time t_0, and the reactivity $\rho(t)$ for $t > t_0$, the future behavior of the reactor can be predicted by solving the system differential equations, and the change in system state is traced as a line in the state plane. If the system is simulated on an analog computer, the neutron density and delayed neutron concentration can be individually displayed on digital meters, individually recorded as a function of time, and plotted on an X-Y recorder. The readings from the two digital meters provide information on the instantaneous state, and the X-Y recorder traces a line in the state plane. The individual recordings provide a parametric display as a function of time.

If two groups of delayed neutrons are used to describe the reactor, then the state space is three dimensional and has the coordinates n, c_1, and c_2. Specifying the values of n, c_1, and c_2 at any time t locates a point in the three dimensional space which describes the state of the reactor. If the reactivity $\rho(t)$ is given, the future behavior of the reactor is traced as a line in the three dimensional state space. The values of n, c_1, and c_2 at any instant are represented by an ordered set of numbers, a vector. The term *vector* is applied to the unique description of a point by an ordered set of numbers and is not intended to imply a directed line segment from the origin. An analog computer simulation will require three digital meters and three recorders. Since three-dimensional X-Y-Z plotters are not available, projections on the X-Y, X-Z, and Y-Z planes may be recorded to afford an indirect visualization of system behavior in the state space. The readings from the three digital meters provide information on the instantaneous state,

and the individual recordings provide a parametric display as a function of time.

With six-groups of delayed neutrons, the state space is seven-dimensional, and seven differential equations are used to describe the system. An ordered set of seven numbers, a vector, describes the system state at any instant of time. An analog computer simulation requires seven digital meters and seven recorders. Twenty-one X-Y plotters would be required to plot all paired combinations of variables if the display method of the three dimensional case was to be extended. In this case, the change in system state cannot be visualized in three dimensional space, but the readings from the seven digital meters specify the instantaneous state and the individual recordings provide the parametric display as a function of time. The ordered set of meter readings gives the numerical value of the system state vector at any instant.

The above discussion may be summarized as follows: n state variables x_1, x_2, x_3,..., x_n are needed to describe completely the behavior of a system described by a set of n first-order differential equations. The set of n state variables can be considered as n components of a vector $\underline{x}$, called the *state vector*. A *state space* is an n-dimensional space in which x_1, x_2,..., x_n are the coordinates. The state of the system at time t can then be represented by a point in an n-dimensional state space. The locus of points in the state space is called a *trajectory*.

Vector-matrix notation is convenient for the representation of system differential equations in state-space analysis. The solution of vector-matrix differential equations is discussed briefly in Appendix A. Detailed treatments of state-space analysis and vector-matrix equations

have been published by: Zadeh and Desoer [87], DeRusso, Roy, and Close [88], Gupta [89], Ogata [90], Timothy and Bona [91], and Chen and Haas [92].

4.2 Six-group representation

Using vector-matrix notation, Eqs. (3.1) and (3.2) can be written:

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \\ \dot{c}_4 \\ \dot{c}_5 \\ \dot{c}_6 \\ \dot{n} \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \beta_1/\ell \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \beta_2/\ell \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \beta_3/\ell \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \beta_4/\ell \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \beta_5/\ell \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \beta_6/\ell \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & [\delta k(t)-\beta]/\ell \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ n \end{bmatrix} \tag{4.1}$$

On defining the generalized state vector:

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ n \end{bmatrix} \tag{4.2}$$

Eq. (4.1) can be rewritten in the form

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) \tag{4.3}$$

where

$$A(t) = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \beta_1/\ell \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \beta_2/\ell \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \beta_3/\ell \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \beta_4/\ell \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \beta_5/\ell \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \beta_6/\ell \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & [\delta k(t)-\beta]/\ell \end{bmatrix} \tag{4.4}$$

As shown in Appendix A, the solution of Eq. (4.3) is given by

$$\underline{x}(t) = \Phi(t,t_0)\underline{x}(t_0) \tag{4.5}$$

where $\Phi(t,t_0)$ is the state transition matrix.

Similarly, Eqs. (3.10) and (3.11) can be written as Eq. (4.3) with

$$\underline{x}(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \\ n(t) \end{bmatrix} \tag{4.6}$$

and

$$A(t) = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \lambda_1 a_1 \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \lambda_2 a_2 \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \lambda_3 a_3 \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \lambda_4 a_4 \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \lambda_5 a_5 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \lambda_6 a_6 \\ \alpha & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha[\rho(t)-1] \end{bmatrix} \tag{4.7}$$

4.3 Six-group prompt-jump representation

The matrix equation corresponding to Eq. (3.29) is

$$\dot{\underline{z}}(t) = A(t)\underline{z}(t) \tag{4.8}$$

with

$$\underline{z}(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \end{bmatrix} \tag{4.9}$$

and

$$A(t) = \frac{1}{1 - \rho(t)} \cdot$$

$$\begin{bmatrix} \lambda_1[\rho(t)+a_1-1] & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 \\ \lambda_2 a_2 & \lambda_2[\rho(t)+a_2-1] & \lambda_2 a_2 & \lambda_2 a_2 & \lambda_2 a_2 & \lambda_2 a_2 \\ \lambda_3 a_3 & \lambda_3 a_3 & \lambda_3[\rho(t)+a_3-1] & \lambda_3 a_3 & \lambda_3 a_3 & \lambda_3 a_3 \\ \lambda_4 a_4 & \lambda_4 a_4 & \lambda_4 a_4 & \lambda_4[\rho(t)+a_4-1] & \lambda_4 a_4 & \lambda_4 a_4 \\ \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5[\rho(t)+a_5-1] & \lambda_5 a_5 \\ \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6[\rho(t)+a_6-1] \end{bmatrix}$$

(4.10)

4.4 One-group representation

The kinetics equations with one group of delayed neutrons, Eqs. (3.32) and (3.33), can be written in matrix notation as Eq. (4.3) with

$$\underline{x}(t) = \begin{bmatrix} z(t) \\ n(t) \end{bmatrix} \tag{4.11}$$

and

$$A(t) = \begin{bmatrix} -\lambda & \lambda \\ \alpha & \alpha[\rho(t) - 1] \end{bmatrix} \tag{4.12}$$

If reactivity is constant with $\rho(t)=\rho$, then the system equation is

$$\dot{\underline{x}}(t) = A\underline{x} \tag{4.13}$$

where

$$A = \begin{bmatrix} -\lambda & \lambda \\ \alpha & \alpha(\rho - 1) \end{bmatrix} \tag{4.14}$$

The solution of Eq. (4.13) can be obtained, as shown in Appendix A, by taking the Laplace transform of Eq. (4.13) to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) \tag{4.15}$$

which can be solved for $\underline{X}(s)$:

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) \tag{4.16}$$

where I is the unit matrix. Equation (4.16) can be written in terms of the Laplace transform of the state transition matrix $\Phi(t)$ as

$$\underline{X}(s) = \Phi(s)\underline{x}(0) \tag{4.17}$$

where $\Phi(s)$, the resolvent matrix, is given by

$$\Phi(s) = [sI - A]^{-1} \tag{4.18}$$

Taking the inverse Laplace transform of Eq. (4.18) results in

$$\Phi(t) = \mathcal{L}^{-1}[sI - A]^{-1} \tag{4.19}$$

where $\Phi(t)$ is the state transition matrix. Using Eq. (4.19), the inverse transform of Eq. (4.17) can be written as

$$\underline{x}(t) = \Phi(t)\underline{x}(0) \tag{4.20}$$

For the matrix defined by Eq. (4.14)

$$sI-A = \begin{bmatrix} s+\lambda & -\lambda \\ -\alpha & s+\alpha(1-\rho) \end{bmatrix} \tag{4.21}$$

and $\Phi(s)$ is given by

$$\Phi(s) = \begin{bmatrix} \frac{s+\alpha(1-\rho)}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} & \frac{\lambda}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} \\ \frac{\alpha}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} & \frac{s+\lambda}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} \end{bmatrix} \tag{4.22}$$

If the root approximations given in Eqs. (3.36) and (3.37) are substituted into Eq. (4.22), then

$$\Phi(s) = \begin{bmatrix} \frac{s+\alpha(1-\rho)}{(s-s_1)(s-s_2)} & \frac{\lambda}{(s-s_1)(s-s_2)} \\ \frac{\alpha}{(s-s_1)(s-s_2)} & \frac{s+\lambda}{(s-s_1)(s-s_2)} \end{bmatrix} \tag{4.23}$$

where

$$s_1 = \lambda\rho/(1-\rho) \tag{4.24}$$

$$s_2 = -\alpha(1-\rho) - \lambda/(1-\rho) \tag{4.25}$$

The state transition matrix is obtained by taking the inverse Laplace transform of Eq. (4.23):

$$\Phi(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \tag{4.26}$$

where

$$\phi_{11}(t) = \frac{[\alpha(1-\rho)^2 + \lambda\rho]e^{s_1 t} + \lambda e^{s_2 t}}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.27}$$

$$\phi_{12}(t) = \frac{\lambda(1-\rho)(e^{s_1 t} - e^{s_2 t})}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.28}$$

$$\phi_{21}(t) = \frac{\alpha(1-\rho)(e^{s_1 t} - e^{s_2 t})}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.29}$$

$$\phi_{22}(t) = \frac{\lambda e^{s_1 t} + [\alpha(1-\rho)^2 + \lambda\rho]e^{s_2 t}}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.30}$$

For t = 0, Eq. (4.26) becomes

$$\Phi(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \tag{4.31}$$

which is one of the properties of the state transition matrix.

The solution for n(t) given by Eq. (4.20), with $\Phi(t)$ given by Eq. (4.26), is identical to the result obtained previously in Eq. (3.38), except that Eq. (4.20) gives, in addition, the solution for the second state variable z(t).

4.5 One-group prompt-jump representation

The system based on the prompt-jump approximation is described by Eqs. (3.39) and (3.40). If the reactivity input is considered, the system equations are augmented by including Eq. (3.50) as follows:

$$\dot{z} = \frac{\lambda\rho z}{1-\rho} \tag{4.32}$$

$$\dot{\rho} = u \tag{4.33}$$

$$n = \frac{z}{1-\rho} \tag{4.34}$$

These equations are expressed in matrix notation as:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, u) \tag{4.35}$$

$$y = h(\underline{x}) \tag{4.36}$$

where

$$\underline{x} = \begin{bmatrix} z \\ \rho \end{bmatrix} \tag{4.37}$$

$$\underline{f} = \begin{bmatrix} f_1(z, \rho) \\ f_2(u) \end{bmatrix} \tag{4.38}$$

$$f_1(z, \rho) = \frac{\lambda \rho z}{1 - \rho} \tag{4.39}$$

$$f_2(u) = u \tag{4.40}$$

$$y = n \tag{4.41}$$

and

$$h(\underline{x}) = h(z, \rho) = \frac{z}{1 - \rho} \tag{4.42}$$

Equation (4.35) is the system nonlinear vector-matrix differential equation, and Eq. (4.36) is the scalar nonlinear measurement equation. The system has a single input u and a single output y.

4.6 Linearization of the system and measurement equations

The system and measurement equations are linearized by considering small perturbations about nominal values of the neutron density n*, normalized precursor level z*, and control input u*. To find the differential equations relating the deviations, expand Eq. (4.35) in a Taylor series

$$\underline{x} = \underline{f}(\underline{x}^*, u^*) + \frac{\partial \underline{f}}{\partial \underline{x}}\bigg|_{x^*} (\underline{x} - \underline{x}^*) + \frac{\partial \underline{f}}{\partial u}\bigg|_{u^*} (u - u^*) + \cdots \tag{4.43}$$

Define

$$\delta \underline{x} = \underline{x} - \underline{x}^* \tag{4.44}$$

$$\delta u = u - u^* \tag{4.45}$$

note that

$$\dot{\underline{x}}^* = \underline{f}(\underline{x}^*, u^*) \tag{4.46}$$

then

$$\delta\dot{\underline{x}} = \dot{\underline{x}} - \dot{\underline{x}}^* \tag{4.47}$$

Finally, substitute Eqs. (4.44), (4.45), and (4.47) into Eq. (4.43), retaining only first-order terms, to obtain

$$\delta\dot{\underline{x}} = \frac{\partial \underline{f}}{\partial \underline{x}}\bigg|_{x^*} \delta\underline{x} + \frac{\partial \underline{f}}{\partial u}\bigg|_{u^*} \delta u \tag{4.48}$$

where

$$\frac{\partial \underline{f}}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{4.49}$$

and

$$\frac{\partial \underline{f}}{\partial u} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \vdots \\ \frac{\partial f_n}{\partial u} \end{bmatrix} \tag{4.50}$$

The measurement equation (4.36) is similarly expanded to obtain

$$y = h(\underline{x}^*) + \frac{\partial h}{\partial \underline{x}}\bigg|_{x^*} (\underline{x} - \underline{x}^*) + \cdots \tag{4.51}$$

which can be written

$$\delta y = \frac{\partial h}{\partial \underline{x}}\bigg|_{x^*} \delta\underline{x} \tag{4.52}$$

where

$$\delta y = y - h(\underline{x}^*) \tag{4.53}$$

and

$$\frac{\partial h}{\partial \underline{x}} = \left[\frac{\partial h}{\partial x_1} \cdots \frac{\partial h}{\partial x_n}\right] \tag{4.54}$$

By defining

$$A = \frac{\partial \underline{f}}{\partial \underline{x}} \tag{4.55}$$

$$D = \frac{\partial \underline{f}}{\partial u} \tag{4.56}$$

and

$$H = \frac{\partial h}{\partial \underline{x}} \tag{4.57}$$

Eqs. (4.48) and (4.52) can be written

$$\dot{\delta \underline{x}} = A\delta \underline{x} + D\delta u \tag{4.58}$$

$$\delta y = H\delta \underline{x} \tag{4.59}$$

The matrices A, D, and H corresponding to Eqs. (4.32), (4.33) and (4.34) are

$$A = \begin{bmatrix} \frac{\lambda\rho^*}{1-\rho^*} & \frac{\lambda z^*}{(1-\rho^*)^2} \\ 0 & 0 \end{bmatrix} \tag{4.60}$$

$$D = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4.61}$$

$$H = \begin{bmatrix} \frac{1}{1-\rho^*} & \frac{z^*}{(1-\rho^*)^2} \end{bmatrix} \tag{4.62}$$

For the particular case in which the reactor is at equilibrium, the nominal values are: $z^* = 1.0$, $\rho^* = 0$, and $u^* = 0$, and the system and measurement equations become

$$\delta\dot{\underline{x}} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix} \delta\underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta u \tag{4.63}$$

$$\delta y = \begin{bmatrix} 1 & 1 \end{bmatrix} \delta\underline{x} \tag{4.64}$$

where

$$\delta\underline{x} = \begin{bmatrix} z - 1.0 \\ \rho \end{bmatrix} = \begin{bmatrix} \delta z \\ \rho \end{bmatrix} \tag{4.65}$$

$$\delta u = u \tag{4.66}$$

and

$$\delta y = n - 1.0 = \delta n \tag{4.67}$$

4.7 Solution of the state-space equations with discrete-time inputs

For a discrete-time input, u is constant for T seconds which can be expressed as

$$u(t) = u_k \qquad kT < t \leq (k + 1)T \tag{4.68}$$

After substituting Eq. (4.68) into Eq. (4.33) and integrating,

$$\rho(t) = \rho(t_k) + u_k(t - t_k) \tag{4.69}$$

which can be written

$$\rho(t) = \rho_k + u_k(t - t_k) \tag{4.70}$$

where ρ_k is the reactivity at the beginning of the interval. Equation (4.70) is substituted into Eq. (4.32) to obtain

$$\dot{z} = \frac{\lambda z[\rho_k + u_k(t - t_k)]}{1 - \rho_k - u_k(t - t_k)} \tag{4.71}$$

which when integrated yields

$$\ell n\left[\frac{z(t)}{z_k}\right] = \frac{\lambda}{u_k} \ell n\left[\frac{1 - \rho_k}{1 - \rho_k - u_k(t - t_k)}\right] - \lambda(t - t_k) \tag{4.72}$$

At $t = t_{k+1}$, Eq. (4.72) is solved for z_{k+1} to obtain

$$z_{k+1} = z_k \exp\left[\frac{\lambda}{u_k} \ell n\left(\frac{1 - \rho_k}{1 - \rho_k - u_k T}\right) - \lambda T\right] \tag{4.73}$$

If $u_k = 0$, then integration of Eq. (4.71) results in

$$\ell n\left[\frac{z(t)}{z_k}\right] = \frac{\lambda \rho_k}{1 - \rho_k}(t - t_k) \tag{4.74}$$

which for $t = t_{k+1}$ yields

$$z_{k+1} = z_k \exp\left[\frac{\lambda \rho_k T}{1 - \rho_k}\right] \tag{4.75}$$

Similarly, ρ_{k+1} is obtained from Eq. (4.70) with $t = t_{k+1}$. Thus

$$\rho_{k+1} = \rho_k + u_k T \tag{4.76}$$

Equations (4.73) and (4.76) provide the finite difference solutions of the system equations at the sampling instants kT. These solutions are exact and do not involve any approximation of the derivative. If $u_k = 0$, Eq. (4.75) is used in place of Eq. (4.73). The corresponding finite difference measurement equation is

$$n_k = \frac{z_k}{1 - \rho_k} \tag{4.77}$$

These finite difference equations may be expressed in matrix notation as follows:

$$\underline{x}_{k+1} = \underline{g}(\underline{x}_k, u_k) \tag{4.78}$$

and

$$y_k = h(\underline{x}_k) \tag{4.79}$$

where

$$\underline{x}_k = \begin{bmatrix} z_k \\ \rho_k \end{bmatrix} \tag{4.80}$$

$$g_1 = z_k \exp\left[\frac{\lambda}{u_k} \ln\left(\frac{1 - \rho_k}{1 - \rho_k - u_k T}\right) - \lambda T\right] \tag{4.81}$$

$$g_2 = \rho_k + u_k T \tag{4.82}$$

and

$$h(\underline{x}_k) = \frac{z_k}{1 - \rho_k} \tag{4.83}$$

4.8 Solution of the linearized equations with discrete-time input

If the delta notation of variable deviation is omitted, Eq. (4.58) can be written

$$\dot{\underline{x}} = A\underline{x} + Du \tag{4.84}$$

When $u = u_k$ for $t_k < t \leq t_{k+1}$, the Laplace transformation of Eq. (4.84) yields

$$s\underline{X}(s) - \underline{x}_k = A\underline{X}(s) + D\frac{u_k}{s} \tag{4.85}$$

which is solved for $\underline{X}(s)$ as follows:

$$\underline{X}(s) = [sI - A]^{-1} \underline{x}_k + \frac{1}{s} [sI - A]^{-1} Du_k \tag{4.86}$$

The solution for $\underline{x}(t)$ is obtained from the inverse transformation of Eq. (4.86) as

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{t_k}^{t} \Phi(t - \tau)Dd\tau \tag{4.87}$$

or

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{0}^{t-t_k} \Phi(\tau)Dd\tau \tag{4.88}$$

At $t = t_{k+1}$

$$\underline{x}_{k+1} = \Phi(T)\underline{x}_k + u_k \int_{0}^{T} \Phi(\tau)Dd\tau \tag{4.89}$$

Equation (4.89) can be written

$$\underline{x}_{k+1} = \Phi\underline{x}_k + Gu_k \tag{4.90}$$

where the control distribution matrix

$$G = \int_{0}^{T} \Phi(\tau)Dd\tau \tag{4.91}$$

As shown in Appendix A

$$\int_{0}^{T} \Phi(\tau)d\tau = A^{-1}[\Phi(T) - I] \tag{4.92}$$

therefore

$$G = A^{-1}[\Phi - I]D \tag{4.93}$$

when A^{-1} exists.

On applying the above procedure to Eq. (4.58), and using the A and D matrices of Eqs. (4.60) and (4.61)

$$\Phi = \begin{bmatrix} \exp\left(\frac{\lambda T\rho^*}{1-\rho^*}\right) & \frac{-z^*\left[1-\exp\left(\frac{\lambda T\rho^*}{1-\rho^*}\right)\right]}{\rho^*(1-\rho^*)} \\ 0 & 1 \end{bmatrix} \tag{4.94}$$

$$G = \begin{bmatrix} \frac{-z^*T}{\rho^*(1-\rho^*)} - \frac{z^*}{\lambda\rho^{*2}}\left[1-\exp\left(\frac{\lambda T\rho^*}{1-\rho^*}\right)\right] \\ T \end{bmatrix} \tag{4.95}$$

If the nominal values correspond to equilibrium conditions, $z^* = 1.0$ and $\rho^* = 0$, and Eqs. (4.94) and (4.95) reduce to

$$\Phi = \begin{bmatrix} 1 & \lambda T \\ 0 & 1 \end{bmatrix} \tag{4.96}$$

$$G = \begin{bmatrix} \frac{\lambda T^2}{2} \\ T \end{bmatrix} \tag{4.97}$$

Substituting Eqs. (4.96) and (4.97) into Eq. (4.90) results in the discrete system equation

$$\underline{x}_{k+1} = \begin{bmatrix} 1 & \lambda T \\ 0 & 1 \end{bmatrix} \underline{x}_k + \begin{bmatrix} \lambda T^2/2 \\ T \end{bmatrix} u_k \tag{4.98}$$

and the discrete output measurement equation obtained using Eq. (4.62) is

$$y_k = [1 \quad 1]\, \underline{x}_k \tag{4.99}$$

Equations (4.98) and (4.99) will be used in deriving the optimal closed loop control law for the regulator problem.

CHAPTER 5

OPTIMAL CONTROL OF NUCLEAR SYSTEMS BY STATE VARIABLE FEEDBACK

5.1 Introduction

Regulation of neutron density in a reactor requires a feedback control law which will compensate for disturbances that occur infrequently and randomly anywhere in time from zero to infinity. If attention is focused on a single disturbance and system noise is neglected, a deterministic regulator problem is formulated.

Dynamic programming is readily applied to linear discrete-time systems, and in the case of a quadratic performance index, leads to the direct calculation of the optimal linear feedback control law. If the performance index is to be minimized over a finite time interval, the feedback control law is a function of time; for an infinite time interval, the feedback control law is stationary and all state variables are fed back through fixed gains. Thus, discrete dynamic programming yields the solution to the reactor regulator problem, if the continuous system is sampled at discrete time intervals.

For a general discussion of dynamic programming, see Bellman [93], Bellman and Kalaba [94], and Dreyfus [95]; and for the dynamic programming solution of discrete-time systems with a quadratic performance index, see Tou [96, p. 45; 97, p. 345] and Lapidus and Luus [98, p. 155].

5.2 Dynamic programming solution of the linear regulator problem

For the discrete-time linear system described by

$$\underline{x}_{k-1} = \Phi \underline{x}_k + G u_k \tag{5.1}$$

and a quadratic performance index of the form

$$I_N = \sum_{k=1}^{N} (\underline{x}_k^T Q \underline{x}_k + c u_{k-1}^2) \tag{5.2}$$

where $\underline{x}_k^T$ is the transpose of $\underline{x}_k$, Q is an n×n positive-definite or semi-definite symmetrical matrix, and c is a positive constant, the optimal control law which minimizes I_N, as shown in Appendix B, is given by

$$u_k = B_{N-k} \underline{x}_k \tag{5.3}$$

where

$$B_j = - \frac{G^T [Q + P_{j-1}] \Phi}{G^T [Q + P_{j-1}] G + c} \tag{5.4}$$

and

$$P_j = [\Phi + G B_j]^T [Q + P_{j-1}][\Phi + G B_j] + c B_j^T B_j \tag{5.5}$$

In Eq. (5.3), the feedback matrix B_{N-k}, a row matrix, is obtained from the iterative solution of Eqs. (5.4) and (5.5). The matrix P_j defined by Eq. (5.5) is n×n and symmetrical. Starting with $P_o = 0$, Eqs. (5.4) and (5.5) yield B_1, P_1, B_2, P_2, ... If the upper limit of summation in Eq. (5.2) is allowed to approach infinity, then B_j converges to a stationary matrix B and Eq. (5.3) reduces to

$$u_k = B \underline{x}_k \tag{5.6}$$

The product of the row matrix B and the state vector $\underline{x}_k$ yields the optimal feedback u_k as indicated in Eq. (5.6).

5.3 Performance indices and constraints

Using Eq. (4.65), the general performance index given by Eq. (5.2) can be written in expanded form as a function of the delayed neutron deviation, reactivity, and reactivity rate:

$$I_N = \sum_{k=1}^{N} (Q_{11}\delta z_k^2 + 2Q_{12}\delta z_k \rho_k + Q_{22}\rho_k^2 + cu_{k-1}^2) \tag{5.7}$$

where

$$\delta z_k = z_k - 1.0 \tag{5.8}$$

and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} \tag{5.9}$$

To regulate the neutron density, a performance index which is a function of the neutron density deviation is defined by:

$$I_N = \sum_{k=1}^{N} \delta n_k^2 \tag{5.10}$$

where

$$\delta n_k = n_k - 1.0 \tag{5.11}$$

Equation (4.99) written in expanded form yields

$$\delta n_k = \delta z_k + \rho_k \tag{5.12}$$

Substitution of Eq. (5.12) into Eq. (5.10) gives

$$I_N = \sum_{k=1}^{N} (\delta z_k^2 + 2\delta z_k + \rho_k^2) \tag{5.13}$$

Comparison of Eq. (5.13) with Eq. (5.7) results in

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{5.14}$$

$$R = 0 \tag{5.15}$$

The Q matrix defined by Eq. (5.14) satisfies the performance index of Eq. (5.10). The optimal control law obtained using this matrix will minimize the sum of the squares of the neutron density deviations at sampling instants.

To reduce the magnitude of the reactivity rate which is applied to correct a disturbance, a penalty term which weights u_{k-1} can be added to Eq. (5.10). Similarly, reactivity can be returned to zero more quickly after a disturbance by adding a penalty term which weights ρ_k. With these additional terms, Eq. (5.10) becomes

$$I_N = \sum_{k=1}^{N} (\delta n_k^2 + a\rho_k^2 + cu_{k-1}^2) \tag{5.16}$$

where a and c are the weighting coefficients. If Eq. (5.12) is substituted into Eq. (5.16), the corresponding matrix

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 1+a \end{bmatrix} \tag{5.17}$$

will result in the minimization of the sum of the squares of the neutron density deviation and the reactivity at the sampling instants.

5.4 Reactor transient response and the performance index

The optimal control law given by Eq. (5.6) is for a linear system as described by Eq. (5.1). Thus, in order to apply the method to the control of a nuclear reactor, the linearized discrete-time Eqs. (4.98) and (4.99) are used, and the Φ and G matrices are substituted into Eqs. (5.4) and (5.5) with $\lambda = 0.31$ and $T = 1$. Arbitrary values are assigned to the a and c weighting coefficients of Eq. (5.16), and the Q matrix of Eq. (5.17) and the coefficient c are substituted into Eqs. (5.4) and (5.5). Equations (5.4) and (5.5) are solved iteratively with $N \to \infty$ to obtain the stationary control law. The Calculation of Feedback Matrix computer program listed in Appendix G iteratively evaluates the B matrix until the difference between successive iterations diminishes to 10^{-7}. Table 5.1 lists the B matrices calculated for nine combinations of a and c.

TABLE 5.1

FEEDBACK MATRIX COEFFICIENTS

a	c	b_1	b_2
0	0	-0.8658008	-1.1341991
0	1	-0.5403229	-0.7918012
0	10	-0.2411739	-0.4557331
1	0	-0.6372618	-1.0987756
1	1	-0.4680735	-0.8534584
1	10	-0.2313746	-0.5005205
10	0	-0.2880492	-1.0446476
10	1	-0.2658030	-0.9705480
10	10	-0.1829017	-0.6938182

The transient response of the reactor is calculated using the nonlinear system Eq. (4.78), the nonlinear measurement Eq. (4.79), and the linear feedback Eq. (5.6). The Calculation of Transient Response computer program listed in Appendix G solves these equations and plots are generated by the Plot Program for Transient Response computer program. Equation (4.81) is unsatisfactory for numerical evaluation with small values of u_k; therefore, a series expansion for Eq. (4.81), derived in Appendix C, is used in the computer program.

Although 1 sec was selected for the control law sampling interval, the system response is evaluated at intermediate sampling instants of 0.1 sec to demonstrate that there is no inter sample ripple.

Figure 5.1 shows the reactor transient response with an initial disturbance of $\rho(0+) = 0.1$ and performance index weighting coefficients $a = 0$ and $c = 0$. At $t = (0-)$, the system is at equilibrium, which corresponds to $\rho(0-) = 0$, $\delta z(0-) = 0$, and $\delta n(0-) = 0$. At $t = (0+)$, a step change of reactivity occurs which gives rise to the prompt jump in neutron density. The control law minimizes the performance index given in Eq. (5.10) by driving the neutron density deviation to essentially zero in 1 sec. The control input at time zero is determined from the product of ρ_0 and b_2 from Table 5.1 or

$$u_0 = -0.1134 \text{ \$/sec} \tag{5.18}$$

The initial control effort is proportional to the reactivity disturbance and inversely proportional to the sampling interval. If the sample interval is doubled, the neutron density deviation is driven to zero in 2 sec and the initial control effort is halved. Similarly, if the sample interval is halved, the initial control effort is doubled.

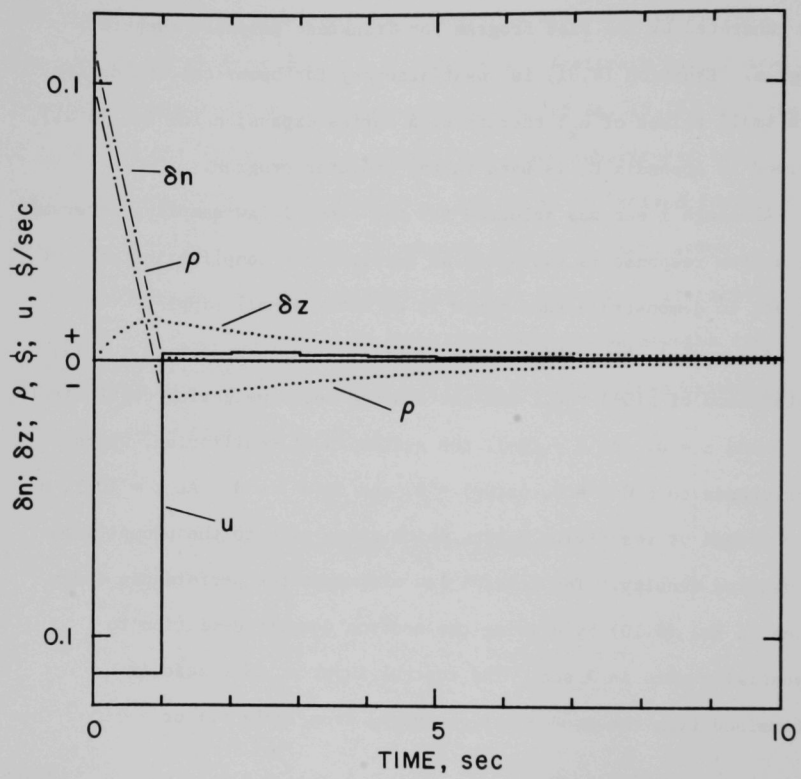

Fig. 5.1. Transient response for a=0, c=0, ρ_0=0.1, δz_0=0.

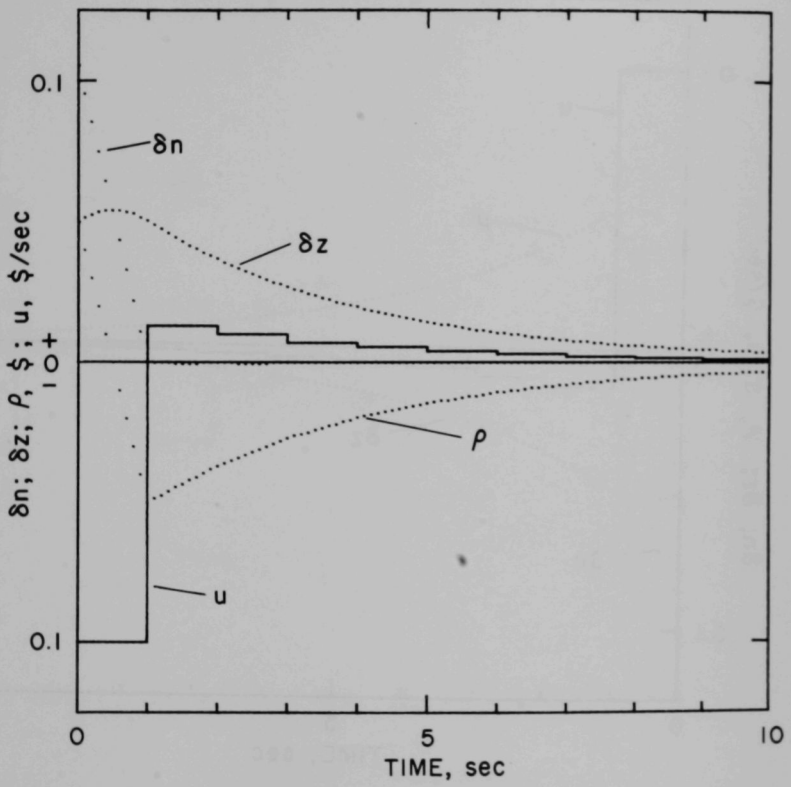

Fig. 5.2. Transient response for a=0, c=0, ρ_0=0.05, δz_0=0.05.

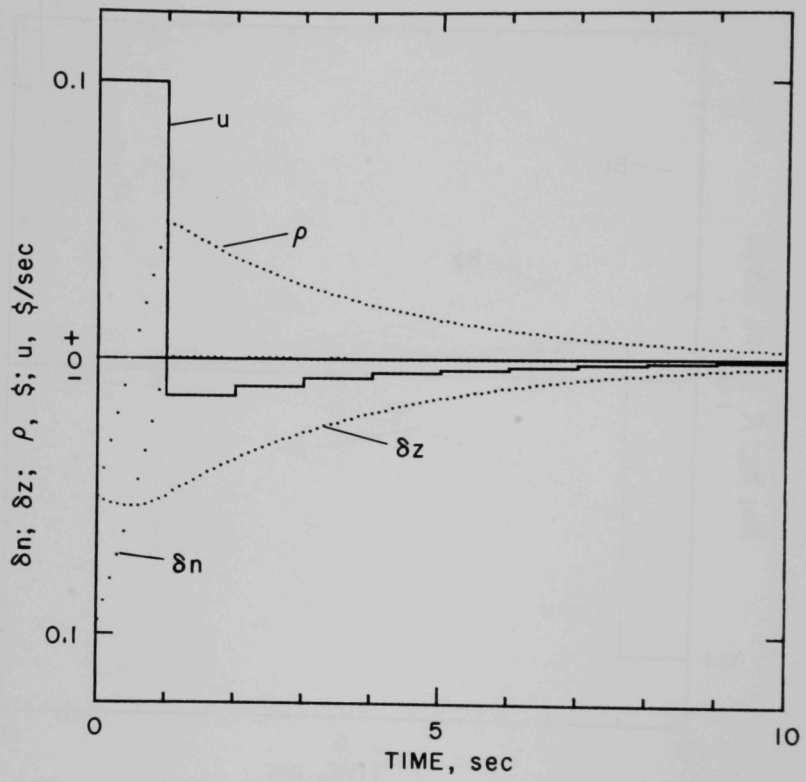

Fig. 5.3. Transient response for a=0, c=0, ρ_0=-0.05, δz_0=-0.05.

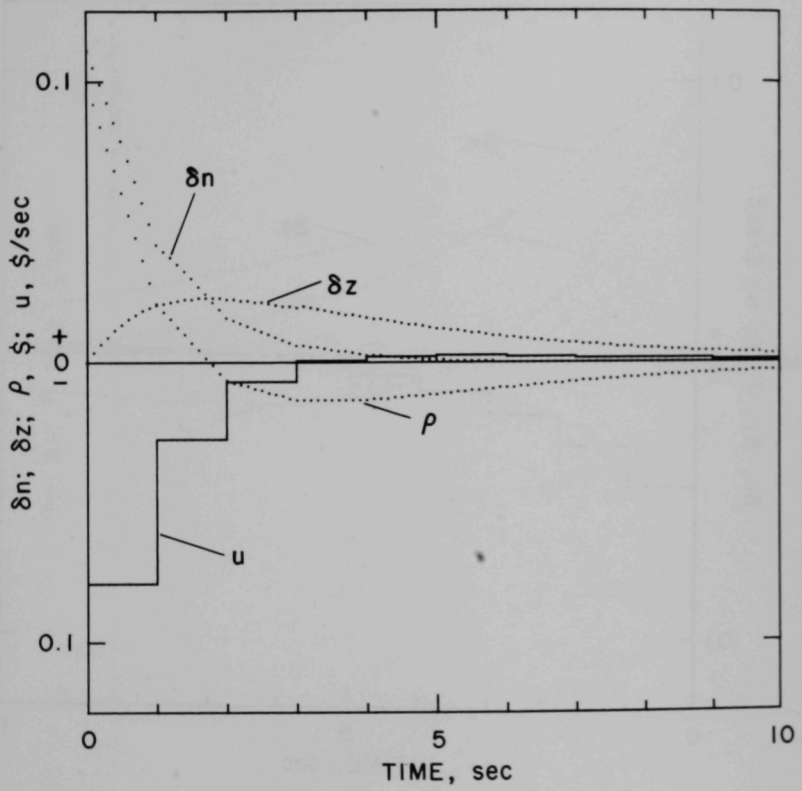

Fig. 5.4. Transient response for a=0, c=1, ρ_0=0.1, δz_0=0.

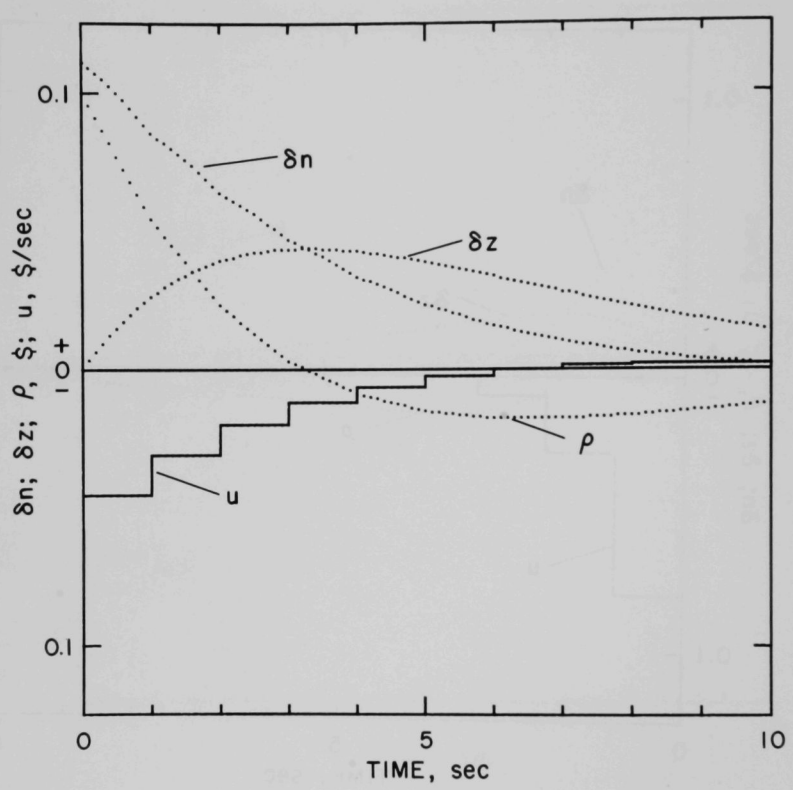

Fig. 5.5. Transient response for a=0, c=10, ρ_0=0.1, δz_0=0.

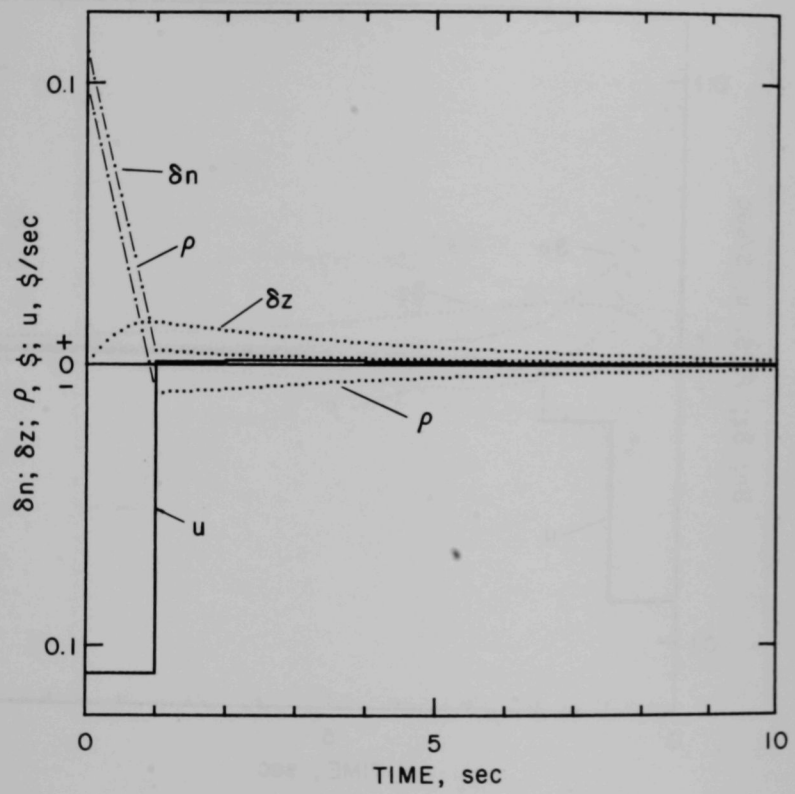

Fig. 5.6. Transient response for a=1, c=0, ρ_0=0.1, δz_0=0.

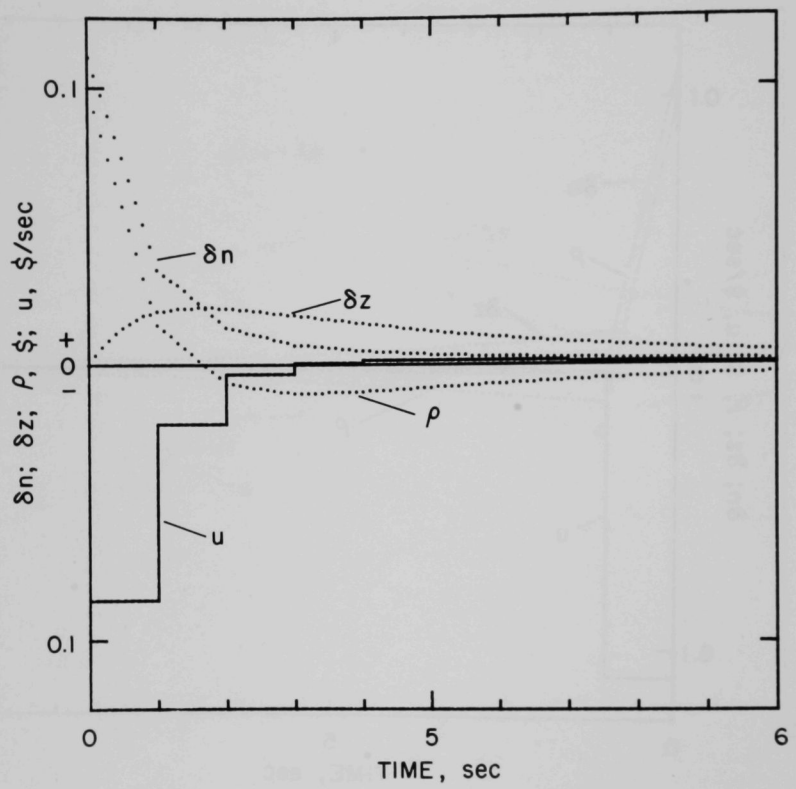

Fig. 5.7. Transient response for a=1, c=1, ρ_0=0.1, δz_0=0.

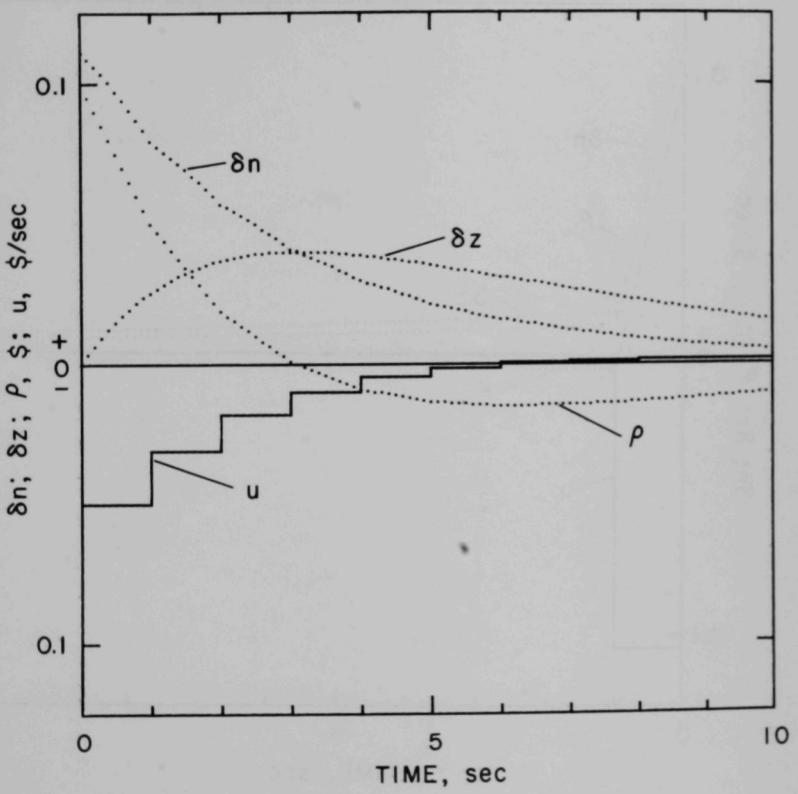

Fig. 5.8. Transient response for a=1, c=10, ρ_0=0.1, δz_0=0.

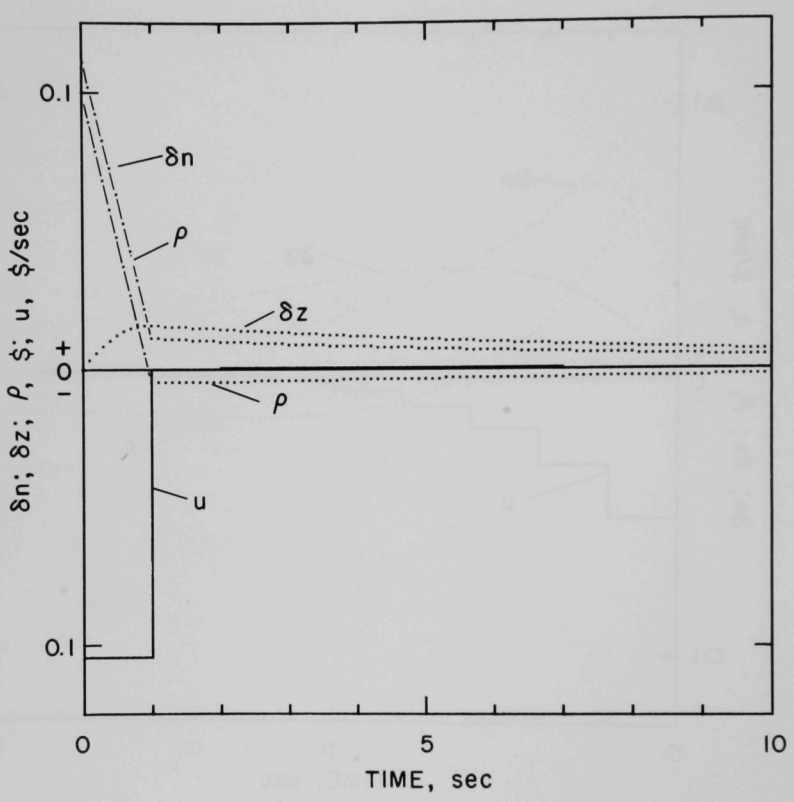

Fig. 5.9. Transient response for a=10, c=0, ρ_0=0.1, δz_0=0.

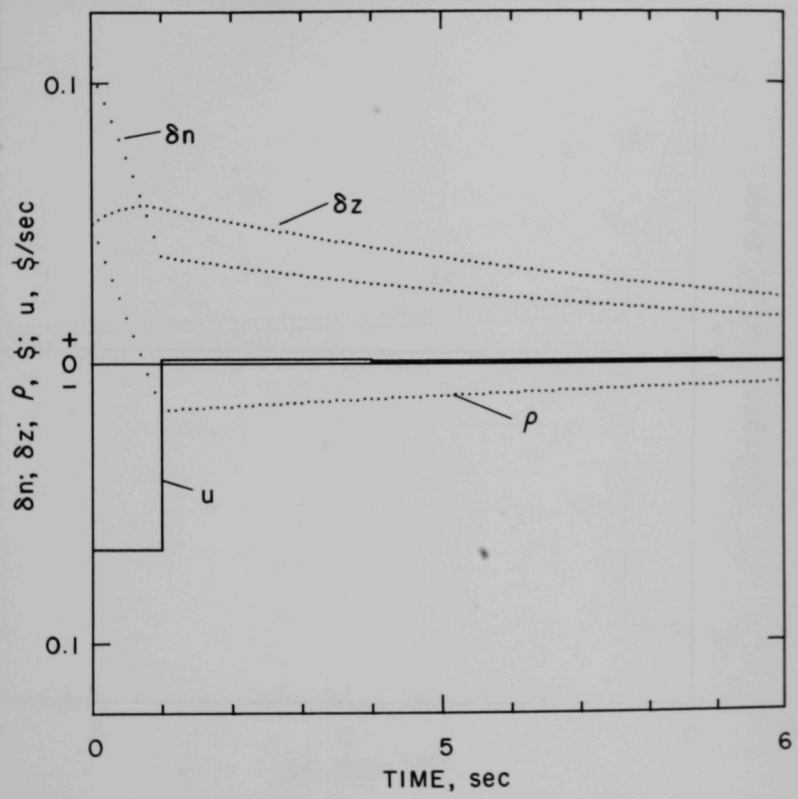

Fig. 5.10. Transient response for a=10, c=0, ρ_0=0.05, δz_0=0.05.

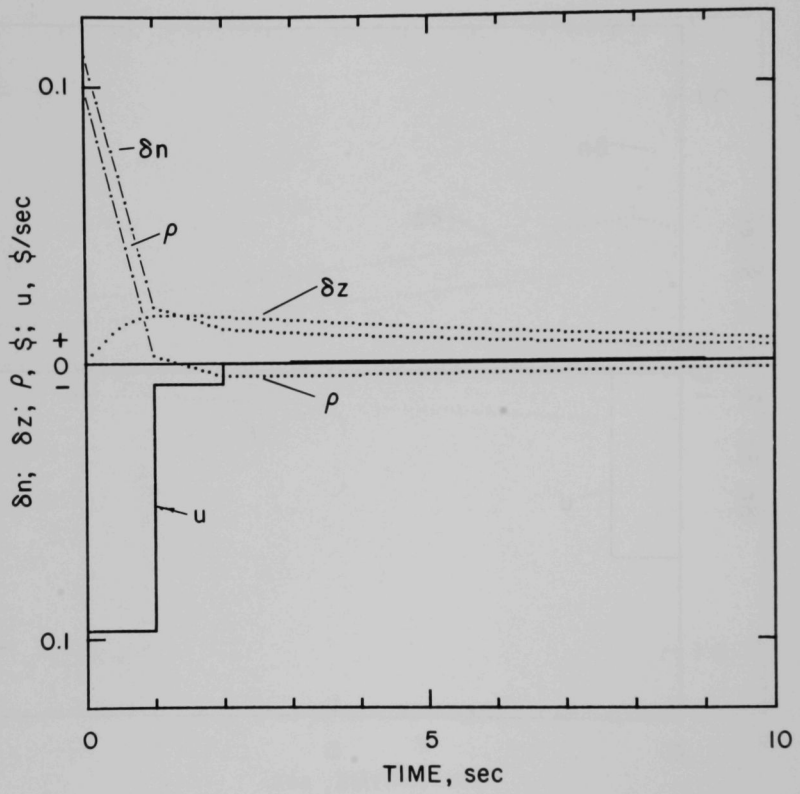

Fig. 5.11. Transient response for a=10, c=1, ρ_0=0.1, δz_0=0.

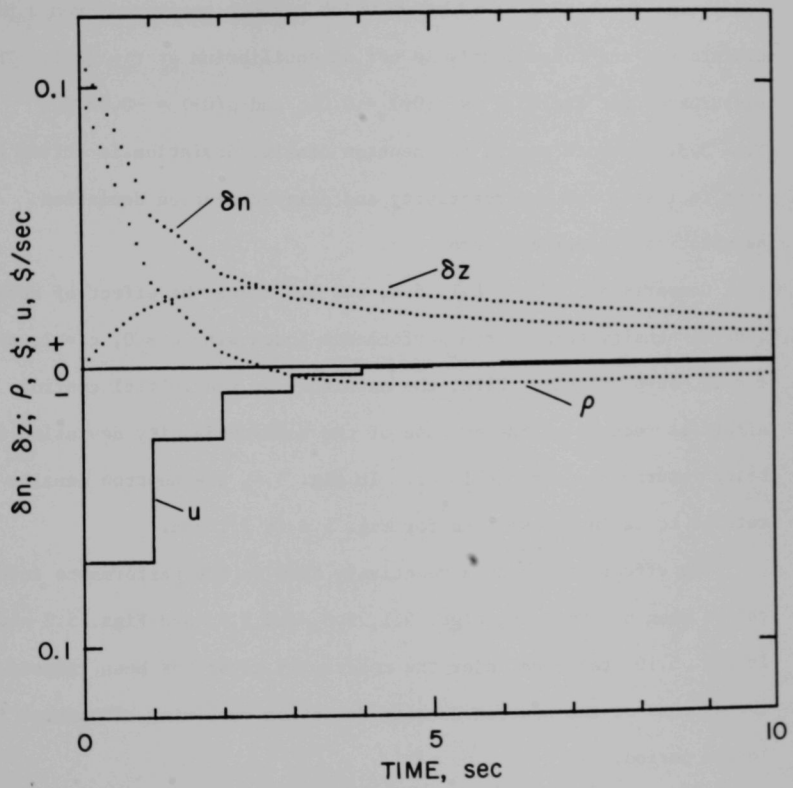

Fig. 5.12. Transient response for a=10, c=10, ρ_0=0.1, δz_0=0.

Figures 5.2 and 5.3 describe system behavior for the same performance index as above, except the initial conditions are different. For Fig. 5.2: $\rho(0-) = 0$, $\delta z(0-) = 0.05$, and $\delta n(0-) = 0.05$; for Fig. 5.3: $\rho(0-) = 0$, $\delta z(0-) = -0.05$ and $\delta n(0-) = -0.05$. These initial conditions correspond to a system which has not recovered from a prior disturbance and consequently is not at equilibrium at $t = (0-)$. The disturbance for Fig. 5.2 is $\rho(0+) = 0.05$, and $\rho(0+) = -0.05$ for Fig. 5.3. In both cases, the neutron density deviation is driven to zero in 1 sec, and the reactivity and delayed neutron deviation asymptotically approach zero.

Comparison of Figs. 5.1, 5.4, and 5.5 shows the effect of adding a control penalty term to the performance index with $c = 0$, $c = 1$, and $c = 10$, respectively. Here, the magnitude of the initial control effort is reduced at the expense of the neutron density deviation not being returned to zero in 1 sec. In Fig. 5.4, the neutron density returns to 1% in 2.6 sec and for Fig. 5.5 in 7.1 sec.

The effect of adding a reactivity term to the performance index can be seen by comparing Figs. 5.1, 5.6, and 5.9, and Figs. 5.2 and 5.10. In Fig. 5.10, the area under the reactivity curve has been reduced at the expense of the neutron density deviation remaining off-normal for a longer period.

Figure 5.7 shows the system behavior with uniform weight assigned to the neutron density deviation, reactivity, and control effort. Figure 12 shows the effect of reducing the weight assigned to the neutron density deviation. Comparison of Fig. 5.7 with Figs. 5.8 and 5.11 shows the effect of increased weight on control effort and reactivity, respectively.

Implementation of the optimal control law given by Eq. (5.6) requires that the system state be known at each sampling instant. In a nuclear reactor, the delayed neutron precursor density and reactivity cannot be measured; consequently, they must be estimated from measurements of the neutron density. An optimal estimator which performs this function is derived in the following chapter.

CHAPTER 6

ESTIMATION OF NUCLEAR SYSTEM STATE VARIABLES

6.1 Introduction

In 1806, Legendre [103] established estimation theory as a mathematical technique with the first publication on least-squares estimation.

In 1960, Kalman [104] solved the Wiener problem for discrete-time systems using state-transition analysis and orthogonal projections, and presented the principle of duality which showed the relationship between stochastic estimation and deterministic control. In a paper on the general theory of control systems [105], he introduced the concepts of controllability and observability. At the joint automatic control conference, Kalman and Bucy [106] extended the method to continuous systems. In a fourth paper, Kalman [107] summarized the contributions of the earlier papers and added a number of theorems and examples.

Ho [108] demonstrated the correspondence between the well-known method of least squares [109] and the optimal-filtering theory of Kalman. He showed that most of the results in linear filtering and prediction theory can be easily derived via a simple lemma on matrix inversion.

Lee [110] in his chapter on optimal estimation discussed: the Wiener filter, the continuous and discrete Wiener-Kalman filter,

least-squares estimation, maximum-likelihood estimation, and the Bayesian approach to estimation.

Ohap and Stubberbud [111] developed a technique for estimating the state of a nonlinear system which combines Kalman's procedure with quasi-linearization. Their technique is not optimal in the strict sense since the linearized dynamic equations are approximations to the nonlinear equations. One advantage of the method is that unlike perturbation equations no *a priori* state of the system must be assumed.

Cox [112] surveyed the methods available for resolving discrete-time estimation problems: Bayesian and weighted least-squares estimation. Least-squares estimation was applied to nonlinear plant and measurement-vector-difference equations. A cost function was formulated which consisted of a linear combination of quadratic forms in errors of an *a priori* estimate, present observation, and plant noise. The constraint due to the plant equation was included by using a Lagrange multiplier, and minimization of the cost function resulted in a pair of nonlinear equations. The latter were solved iteratively to obtain the optimal estimate. Linearized Kalman filtering was indicated as being equivalent to a single iteration.

An alternate method of solving a cost function also was described. This method results in a two-point boundary value problem which is solved by successive approximations. M-step smoothing was introduced as a method to alleviate the difficulty of computer memory requirements increasing linearly with the number of observations. It was pointed out that for systems with no plant noise, the linearized Kalman filter is asymptotically open loop because the filter gain approaches zero.

Mowery [113] presented an optimal filter solution for a plant described by a nonlinear-vector-differential equation and a nonlinear-vector-measurement equation. The nonlinear plant equations were linearized about a nominal solution and a set of difference equations was obtained. The nonlinear measurement equation was similarly linearized. A criterion function was formulated which consisted of a linear combination of quadratic forms in errors of an *a priori* estimate and present observation. Minimizing the criterion function with respect to the new estimate resulted in a set of nonlinear normal equations. The solution of the linearized plant equation was used to derive the relationship between the *a priori* and *a posteriori* error weighting matrices. An iteration scheme was proposed to reduce the disparity between the nominal state vector and the true value.

Deutsch [114] in a chapter on differential equation techniques for linear filtering and prediction included the Kalman-Bucy method, discrete-time estimation, nonstationary estimation, and Bayes'-estimation formulation.

Sridhar and Pearson [115] presented an approximate solution to the problem of digital sequential, least-squares estimation of states and parameters in nonlinear processes. Observations were assumed to be linear, and a cost function was formulated which consisted of the sum of a linear combination of quadratic forms in errors of the state vector estimates and observations. A Lagrange multiplier vector was used to add the plant constraint to the cost function. Minimization of the cost function resulted in a nonlinear two-point boundary value problem which was solved by invariant imbedding to obtain the filter equations. An example was presented for the solution of a system represented by a

nonlinear differential equation. Integration was used to obtain the solution of the nonlinear plant equation at discrete time intervals. Similarly, the plant variational equation was integrated to obtain the value of the derivative of the plant nonlinear difference equation with respect to the state vector.

Peschon, et al., [116, p. 70; 117, p. 6-8] derived an extended Kalman filter by linearizing the process and measurement nonlinear finite difference equations around the last estimate.

Phillips [118] used least-squares theory to formulate a cost function for a discrete-time nonlinear plant and nonlinear measurement system. A Lagrange multiplier was used to include the plant equation constraint. The two-point boundary value problem which results from the minimization of the cost function was solved by invariant imbedding to obtain the filter equations. The resulting filter equations extend the earlier work of Sridhar and Pearson [115] by considering a nonlinear measurement equation.

Sorenson [119] investigated optimal estimation and control policies for discrete-time, stochastic, dynamic systems. Perturbation techniques were applied, terms higher than first order were retained, and the estimation and control policies were determined using the Bayesian approach. In Reference 120 he summarized Kalman filtering techniques. A system consisting of a nonlinear plant and nonlinear measurement equation was analyzed by using linear perturbation equations with the coefficients evaluated at nominal values.

Sage and Masters [121] showed the relationship between least-squares-curve fitting and optimum filtering for linear systems. The Kalman-Bucy solution to the Wiener filtering problem was presented using

least-squares techniques and the Bayesian rule. Relationships between least-squares, minimum-variance, and minimum-mean-squared-error estimates also were described.

Irwin [122] investigated estimation for discrete-time systems. The Bayesian, maximum likelihood, conditional expectation, dynamic programming, orthogonal projection, and two-point boundary value problem approaches were used to derive the Kalman filter equations. The solutions for nonlinear systems consisted of: the Kalman filter linearized about the present estimate; iterative solution of the equations resulting from the dynamic programming approach; and the two-point boundary value problem approach. A new approach was presented for the nonlinear estimator which utilized a performance index consisting of the logarithm of the conditional probability of the present estimate based on a set of measurements. Minimization of the performance index resulted in a set of nonlinear algebraic equations whose solution yields the optimal estimate.

Pearson [123] extended the the work of Sridhar and Pearson [115] to include nonlinear measurements. His result was the same as that of Phillips [118].

Liebeldt [124] included a chapter on linear discrete dynamic estimation and derived the Kalman filter.

Sage [125] devoted chapters to optimum state estimation in linear stationary systems, optimum filtering for nonstationary continuous systems, and least-squares curve fitting and state estimation in discrete linear systems.

Of the estimation methods outlined above, the iterative procedure presented by Cox comes closest to providing the solution for the

deterministic nuclear system state estimator. The filter gain for a deterministic system with the fastest observation scheme is different from the filter gain derived for a stochastic system, so a sequential development of a nuclear system state estimator is presented starting with discrete-time equations and a linear Kalman estimator. Although the estimator derivation is based on discrete-time difference equations, integration is introduced into the estimator to make the method directly applicable to a plant described by a nonlinear vector differential equation and nonlinear measurement equation.

6.2 Kalman filter

For the discrete-time linear system described by

$$\underline{x}_{k+1} = \Phi \underline{x}_k \tag{6.1}$$

and

$$y_k = H\underline{x}_k \tag{6.2}$$

the fastest observation scheme is uniquely determined by

$$\underline{\hat{x}}_{k+1} = \Phi \underline{\hat{x}}_k + \underline{f}_1 (y_k - \hat{y}_k) \tag{6.3}$$

where $\underline{\hat{x}}_k$ is the estimate of the system state at instant k; $\underline{f}_1$ is the first element of the dual basis of $\underline{f}_1^*, \ldots, \underline{f}_n^*$, where

$$\underline{f}_i^* = (\Phi^T)^{-i} H^T \tag{6.4}$$

and

$$\hat{y}_k = H\underline{\hat{x}}_k \tag{6.5}$$

If the dual basis of F is

$$F^* = [\underline{f}_1^*, \ldots, \underline{f}_n^*] \tag{6.6}$$

then

$$F = (F^{*T})^{-1} \tag{6.7}$$

or

$$F = [\underline{f}_1, \ldots, \underline{f}_n] \tag{6.8}$$

For the discrete-time linear reactor Eqs. (4.98) and (4.99),

$$\underline{f}_1^* = \begin{bmatrix} 1 & 0 \\ \lambda T & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - \lambda T \end{bmatrix} \tag{6.9}$$

$$\underline{f}_2^* = \begin{bmatrix} 1 & 0 \\ \lambda T & 2 \end{bmatrix}^{-2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - 2\lambda T \end{bmatrix} \tag{6.10}$$

$$F^* = \begin{bmatrix} 1 & 1 \\ 1 - \lambda T & 1 - 2\lambda T \end{bmatrix} \tag{6.11}$$

$$F = \begin{bmatrix} (2\lambda T - 1)/\lambda T & 1 - 1/\lambda T \\ 1/\lambda T & 1/\lambda T \end{bmatrix} \tag{6.12}$$

and

$$\underline{f}_1 = \begin{bmatrix} (2\lambda T - 1)/\lambda T \\ 1/\lambda T \end{bmatrix} \tag{6.13}$$

Thus the estimator described by Eq. (6.3) with the $\underline{f}_1$ of Eq. (6.13) will generate an optimal estimate of the system state, after a disturbance, using a maximum of two output measurements. In general, for an nth-order system, the optimal estimate is obtained using a maximum of n output measurements.

As shown in Figs. 5.1 through 5.12, the reactivity and delayed neutron deviation do not correspond to the nominal values of $z^* = 1.0$

and $\rho^* = 0$ which were assumed in deriving Eqs. (4.98) and (4.99); therefore, it would be better to use Eqs. (4.62) and (4.94) to evaluate the H and Φ matrices, except the nominal values must be known. The extended Kalman filter method uses the last estimate as the nominal value, which is satisfactory if successive values do not change rapidly. As will be shown later, there is a very large change in nominal values after a reactivity disturbance; thus the extended Kalman filter fails to provide the correct estimates of the reactor state. The question of unknown nominal values is resolved by using the iteration method proposed by Cox [112].

6.3 Linear estimation by matrix inversion

For the dynamic system described by

$$\underline{x}_{k+1} = \phi(k+1, k)\underline{x}_k \tag{6.14}$$

and

$$y_k = H_k\underline{x}_k \tag{6.15}$$

assume k output measurements have been made which are related as follows:

$$\begin{aligned} y_1 &= H_1\underline{x}_1 \\ y_2 &= H_2\underline{x}_2 \\ &\vdots \\ y_k &= H_k\underline{x}_k \end{aligned} \tag{6.16}$$

These measurements can be referred to $\underline{x}_k$ by using Eq. (6.14) with $\underline{x}_j = \phi(j,k)\underline{x}_k$, and written in composite form. Thus

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{bmatrix} = \begin{bmatrix} H_1 \Phi(1,k)\underline{x}_k \\ H_2 \Phi(2,k)\underline{x}_k \\ \cdot \\ \cdot \\ \cdot \\ H_k \Phi(k,k)\underline{x}_k \end{bmatrix} \tag{6.17}$$

Equation (6.17) can be partitioned to yield

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{bmatrix} = \begin{bmatrix} H_1\Phi(1,k) \\ \text{--------} \\ H_2\Phi(2,k) \\ \text{--------} \\ \cdot \\ \cdot \\ \cdot \\ \text{--------} \\ H_k\Phi(k,k) \end{bmatrix} \underline{x}_k \tag{6.18}$$

and written more compactly as

$$\underline{y}_k = \underline{H}_k \underline{x}_k \tag{6.19}$$

where $\underline{y}_k$ is the vector of output measurements, and $\underline{H}_k$ is the composite matrix shown in Eq. (6.18).

The fastest observation scheme is obtained when the number of output measurements is equal to the order of the system. With k = n, Eq. (6.19) can be solved for $\underline{x}_k$ by left-multiplying by $\underline{H}_k^T$,

$$\underline{H}_k^T \underline{y}_k = \underline{H}_k^T \underline{H}_k \underline{x}_k \tag{6.20}$$

and by $[\underline{H}_k^T \underline{H}_k]^{-1}$, to finally obtain

$$\underline{x}_k = [\underline{H}_k^T \underline{H}_k]^{-1} \underline{H}_k^T \underline{y}_k \tag{6.21}$$

which gives the optimal estimate of the state at instant k for a set of k measurements.

A sequential form for estimation can be obtained by writing Eq. (6.18) as follows:

$$\begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} = \begin{bmatrix} \underline{H}_{k-1}\underline{x}_{k-1} \\ \hline H_k \underline{x}_k \end{bmatrix} \tag{6.22}$$

where $\underline{y}_{k-1}$ is a vector of k-1 output measurements, and $\underline{H}_{k-1}$ is a composite matrix defined by the first k-1 elements in Eq. (6.18). The vector $\underline{x}_{k-1}$ can be written in terms of $\underline{x}_k$. With simplified notation

$$\underline{x}_{k-1} = \Phi^{-1}(k,k-1)\underline{x}_k = \Phi_{k-1}^{-1}\underline{x}_k \tag{6.23}$$

and substitution of Eq. (6.23) into (6.22) yields

$$\begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} = \begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix} \underline{x}_k \tag{6.24}$$

Solution of Eq. (6.24) for $\underline{x}_k$ is obtained by multiplication by the inverse matrix:

$$\underline{x}_k = \begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix}^{-1} \begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} \tag{6.25}$$

and comparison with Eq. (6.21) shows that

$$\begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix}^{-1} = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T} \mid H_k^T] \tag{6.26}$$

where $(\Phi_{k-1}^{-1})^T = \Phi_{k-1}^{-T}$. Therefore

$$\underline{x}_k = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{y}_{k-1} + H_k^T y_k] \tag{6.27}$$

Equation (6.27) can be written in terms of $\underline{x}_{k-1}$ by substituting for $\underline{y}_{k-1}$ from Eq. (6.22) to obtain

$$\underline{x}_k = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\underline{x}_{k-1} + H_k^T y_k] \tag{6.28}$$

Equation (6.28) can be rearranged into a form containing an error correction term by multiplying both sides of the equation with the result that

$$[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]\underline{x}_k = \Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\underline{x}_{k-1} + H_k^T y_k \tag{6.29}$$

If the term $H_k^T H_k \Phi_{k-1}\underline{x}_{k-1}$ is added and subtracted to the right hand side, Eq. (6.29) can be written as follows:

$$\begin{aligned}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]\underline{x}_k = {} & \Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1}\Phi_{k-1}\underline{x}_{k-1} \\ & + H_k^T H_k \Phi_{k-1}\underline{x}_{k-1} - H_k^T H_k \Phi_{k-1}\underline{x}_{k-1} \\ & + H_k^T y_k \end{aligned} \tag{6.30}$$

Multiplication of both sides of Eq. (6.30) finally yields

$$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} + [\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}H_k^T[y_k - H_k\Phi_{k-1}\underline{x}_{k-1}] \tag{6.31}$$

which is in the form of Kalman's Eq. (6.3), except that $\underline{x}_k$ is generated with the y_k output sample. This is the filtering equation.

If $\underline{\hat{x}}_{k-1}$ and $\underline{\hat{x}}_k$ are defined as the optimal filter outputs, then an optimal estimate is predicted by using the transition matrix to yield

$$\underline{\bar{x}}_k = \Phi_{k-1}\underline{\hat{x}}_{k-1} \tag{6.32}$$

where $\underline{\bar{x}}_k$ is the predicted value of $\underline{\hat{x}}_k$ obtained using the y_{k-1} measurement. Equation (6.32) is the prediction equation.

If in Eq. (6.31), $\underline{x}_{k-1}$ and $\underline{x}_k$ are replaced by $\underline{\hat{x}}_{k-1}$ and $\underline{\hat{x}}_k$, respectively, and Eq. (6.32) is used, then Eq. (6.31) becomes

$$\underline{\hat{x}}_k = \underline{\bar{x}}_k + [\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T[y_k - H_k\underline{\bar{x}}_k] \tag{6.33}$$

The system state at t = (k + 1)T is predicted from

$$\underline{\bar{x}}_{k+1} = \Phi_k\underline{\hat{x}}_k \tag{6.34}$$

If both sides of Eq. (6.33) are multiplied by Φ_k and Eq. (6.34) is substituted for the left side

$$\underline{\bar{x}}_{k+1} = \Phi_k\underline{\bar{x}}_k + \Phi_k[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T[y_k - H_k\underline{\bar{x}}_k] \tag{6.35}$$

which is Kalman's formula with

$$\underline{f}_1 = \Phi_k[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T \tag{6.36}$$

Equation (6.33) yields the optimal estimate of the system at instant k using an *a priori* estimate $\underline{\bar{x}}_k$ and an error correction term based on measurement y_k. $\underline{\hat{x}}_k$ is the *a posteriori* estimate. A new *a priori* estimate is generated using Eq. (6.34).

Equation (6.35) generates a new *a priori* estimate from the old *a priori* estimate with an error correction term based on the current measurement.

6.4 Linear estimation by least-squares minimization

The least-squares estimate of $\underline{x}_k$ is obtained by minimizing the following cost function:

$$J = (y_1 - H_1\underline{x}_1)^2 + (y_2 - H_2\underline{x}_2)^2 + \ldots + (y_k - H_k\underline{x}_k)^2 \tag{6.37}$$

subject to

$$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} \tag{6.38}$$

Equation (6.37) can be written using Eq. (6.22) as follows:

$$J = [\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}] + (y_k - H_k\underline{x}_k)^2 \tag{6.39}$$

The constraint defined by Eq. (6.38) can be included by defining a vector Lagrangian multiplier λ and augmenting Eq. (6.39). The new cost function is

$$J = [\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}] + (y_k - H_k\underline{x}_k)^2 + \lambda^T[\underline{x}_k - \Phi_{k-1}\underline{x}_{k-1}] \tag{6.40}$$

Setting the gradient of J with respect to $\underline{x}_{k-1}$, $\underline{x}_k$, and λ, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = -2[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T H_{k-1} - \lambda^T\Phi_{k-1} = 0 \tag{6.41}$$

and

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - H_k\underline{x}_k]^T H_k + \lambda^T = 0 \tag{6.42}$$

and

$$\frac{\partial J}{\partial \lambda} = [\underline{x}_k - \Phi_{k-1}\underline{x}_{k-1}]^T = 0 \tag{6.43}$$

Equation (6.43) is the original system Eq. (6.38). Equation (6.42) is solved for λ^T to obtain

$$\lambda^T = -2[y_k - H_k \underline{x}_k]H_k \tag{6.44}$$

and λ^T is eliminated from Eq. (6.41) with the result that

$$\underline{H}_{k-1}^T y_{k-1} - \underline{H}_{k-1}^T \underline{H}_{k-1} \underline{x}_{k-1} = \Phi_{k-1}^T H_k^T y_k - \Phi_{k-1}^T H_k^T H_k \underline{x}_k \tag{6.45}$$

which on multiplication by Φ_{k-1}^{-T} yields

$$\Phi_{k-1}^{-T} \underline{H}_{k-1}^T y_{k-1} - \Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{H}_{k-1} \underline{x}_{k-1} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.46}$$

If $\underline{x}_{k-1}$ is replaced by using Eq. (6.38), then Eq. (6.46) can be written

$$\Phi_{k-1}^{-T} \underline{H}_{k-1}^T y_{k-1} - \Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{H}_{k-1} \Phi_{k-1}^{-1} \underline{x}_k = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.47}$$

which, in turn, can be rearranged in the form of Eq. (6.27) by using the matrix inverse.

An alternate cost function can be defined [112, 113, 122]:

$$J = [\underline{H}_\alpha (\underline{x}_{k-1} - \underline{\alpha})]^2 + (y_k - H_k \underline{x}_k)^2 + \lambda^T [\underline{x}_k - \Phi_{k-1} \underline{x}_{k-1}] \tag{6.48}$$

where $\underline{\alpha}$ is the previous estimate. Setting the gradients of J with respect to $\underline{x}_{k-1}$ and $\underline{x}_k$, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = 2[\underline{H}_\alpha (\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_\alpha - \lambda^T \Phi_{k-1} = 0 \tag{6.49}$$

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - H_k \underline{x}_k]^T H_k + \lambda^T = 0 \tag{6.50}$$

and elimination of λ results in

$$[\underline{H}_{\alpha}(\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_{\alpha} = [y_k - H_k \underline{x}_k]^T H_k \Phi_{k-1} \tag{6.51}$$

Transposing Eq. (6.51) and multiplying by Φ_{k-1}^{-1} leads to

$$\Phi_{k-1}^{-1} \underline{H}_{\alpha}^T \underline{H}_{\alpha} \underline{x}_{k-1} - \Phi_{k-1}^{-1} \underline{H}_{\alpha}^T \underline{H}_{\alpha} \underline{\alpha} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.52}$$

Equation (6.38) is used to eliminate $\underline{x}_{k-1}$ with the result that

$$\Phi_{k-1}^{-1} \underline{H}_{\alpha}^T \underline{H}_{\alpha} \Phi_{k-1}^{-1} \underline{x}_k - \Phi_{k-1}^{-1} \underline{H}_{\alpha}^T \underline{H}_{\alpha} \underline{\alpha} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.53}$$

Equation (6.53) can be rearranged in the form of Eq. (6.29) which is obtained by the matrix inverse.

6.5 Nonlinear estimation by least-squares minimization and iteration

For the nonlinear plant defined by

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}) \tag{6.54}$$

and the nonlinear measurement equation

$$y_k = h(\underline{x}_k) \tag{6.55}$$

an optimal estimate of the system state can be obtained by minimizing the following cost function:

$$J = [\underline{H}_{\alpha}(\underline{x}_{k-1} - \underline{\alpha})]^2 + [y_k - h(\underline{x}_k)]^2 + \lambda^T [\underline{x}_k - \underline{f}(\underline{x}_{k-1})] \tag{6.56}$$

where $\underline{\alpha}$ is the previous estimate. Setting the gradients of J with respect to $\underline{x}_{k-1}$ and $\underline{x}_k$, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = 2[\underline{H}_{\alpha}(\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_{\alpha} - \lambda^T F_{k-1} = 0 \tag{6.57}$$

and

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - h(\underline{x}_k)]^T H_k + \lambda^T = 0 \tag{6.58}$$

where

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1})}{\partial \underline{x}_{k-1}} \tag{6.59}$$

and

$$H_k = \frac{\partial h(\underline{x}_k)}{\partial \underline{x}_k} \tag{6.60}$$

Eliminating λ^T from Eqs. (6.57) and (6.58) results in

$$[\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_\alpha = [y_k - h(x_k)]^T H_k F_{k-1} \tag{6.61}$$

and after transposing, Eq. (6.61) becomes

$$\underline{H}_\alpha^T \underline{H}_\alpha (\underline{x}_{k-1} - \underline{\alpha}) = F_{k-1}^T H_k^T [y_k - h(x_k)] \tag{6.62}$$

Equations (6.54) and (6.62) must be satisfied for J to be a minimum.

The estimation process may be interpreted as follows. Given the last estimate $\underline{\alpha}$ based on a measurement y_{k-1}, a revised estimate $\underline{x}_{k-1}$ is made which must satisfy

$$\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha}) = 0 \tag{6.63}$$

This revised estimate is used in Eq. (6.54) to obtain an estimate of $\underline{x}_k$, which, in turn, must satisfy

$$y_k - h(\underline{x}_k) = 0 \tag{6.64}$$

Nonlinear Eqs. (6.54) and (6.62) can be solved by iteration by using a first-order Taylor expansion:

$$\underline{x}_k^{i+1} = \underline{f}(\underline{x}_{k-1}^{i+1}) \approx \underline{f}(x_{k-1}^i) + F_{k-1}\,(x_{k-1}^{i+1} - x_{k-1}^i) \tag{6.65}$$

$$h(\underline{x}_k^{i+1}) \approx h(\underline{x}_k) + H_k\,(x_k^{i+1} - x_k^i) \tag{6.66}$$

where the superscripts identify the iteration sequence. Equation (6.62) at the i+1 iteration is

$$\underline{H}_{\alpha\alpha}^{T}\underline{H}_{\alpha\alpha}(\underline{x}_{k-1}^{i+1} - \underline{\alpha}) = F_{k-1}^{T}H_{k}^{T}[y_{k} - h(x_{k}^{i+1})] \tag{6.67}$$

and substitution of Eqs. (6.65) and (6.66) into Eq. (6.67) results in

$$\underline{H}_{\alpha\alpha}^{T}\underline{H}_{\alpha\alpha}(\underline{x}_{k-1}^{i+1} - \underline{\alpha}) = F_{k-1}^{T}H_{k}^{T}[y_{k} - h(x_{k}^{i}) + H_{k}\underline{x}_{k}^{i} - H_{k}\underline{f}(\underline{x}_{k-1}^{i})$$

$$- H_{k}F_{k-1}\underline{x}_{k-1}^{i+1} + H_{k}F_{k-1}\underline{x}_{k-1}^{i}] = 0 \tag{6.68}$$

The term $\underline{H}_{\alpha\alpha}^{T}\underline{H}\,\underline{x}_{k-1}^{i}$ is added and subtracted to Eq. (6.68) to obtain

$$[\underline{H}_{\alpha\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]\underline{x}_{k-1}^{i+1} = [\underline{H}_{\alpha\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]\underline{x}_{k-1}^{i}$$

$$+ F_{k-1}^{T}H_{k}^{T}[y_{k} - h(\underline{x}_{k}^{i}) + H_{k}[\underline{x}_{k}^{i} - \underline{f}(\underline{x}_{k-1}^{i})]\}$$

$$+ \underline{H}_{\alpha\alpha}^{T}\underline{H}_{\alpha\alpha}(\underline{\alpha} - \underline{x}_{k-1}^{i}) \tag{6.69}$$

Multiplication of Eq. (6.69) by the inverse matrix yields

$$\underline{x}_{k-1}^{i+1} = \underline{x}_{k-1}^{i} + [\underline{H}_{\alpha\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]^{-1}\{F_{k-1}^{T}H_{k}^{T}[y_{k} - h(\underline{x}_{k}^{i})$$

$$+ H_{k}(\underline{x}_{k}^{i} - \underline{f}(\underline{x}_{k-1}^{i}))] + \underline{H}_{\alpha\alpha}^{T}\underline{H}_{\alpha\alpha}(\underline{\alpha} - \underline{x}_{k-1}^{i})\} \tag{6.70}$$

and the i+1 estimate for $\underline{x}_{k}$ is obtained from

$$\underline{x}_{k}^{i+1} = \underline{f}(x_{k-1}^{i}) + F_{k-1}(x_{k-1}^{i+1} - x_{k-1}^{i}) \tag{6.71}$$

Equations (6.70) and (6.71) are the estimator equations for a system consisting of a nonlinear plant with a nonlinear measurement. The iteration sequence is started by selecting

$$\underline{x}_{k-1}^{1} = \underline{\alpha} \tag{6.72}$$

and

$$\underline{x}_k^1 = \underline{f}(\underline{\alpha}) \tag{6.73}$$

With each iteration, the H_k and F_{k-1} matrices are re-evaluated and a new matrix inverse is calculated. A matrix inversion lemma applied to stochastic systems to eliminate the inversion is not applicable to Eq. (6.70) [125, p. 276].

The term $\underline{H}_{\alpha}(\underline{\alpha} - \underline{x}_{k-1}^i)$ which appears on the right hand side of Eq. (6.70) is identically equal to zero throughout the iteration sequence. A proof that

$$\underline{H}_{\alpha}(\underline{\alpha} - \underline{x}_{k-1}^i) = \underline{0} \tag{6.74}$$

is given in Appendix D.

6.6 Nonlinear estimation of continuous systems with discrete time measurements

The nonlinear estimator defined by Eqs. (6.70) and (6.71) was derived for a system described by nonlinear difference Eqs. (6.54) and (6.55).

For a plant described by

$$\dot{\underline{x}} = \underline{g}(\underline{x}) \tag{6.75}$$

the value of $\underline{x}_k$ is obtained by integration:

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}) = \underline{x}_{k-1} + \int_0^T \underline{g}(\underline{x})dt \tag{6.76}$$

The estimator requires $\partial \underline{f}(\underline{x}_{k-1})/\partial \underline{x}_{k-1}$ which is obtained by integrating the solution of the plant variational equation. The variational equation is given by

$$\dot{\delta \underline{x}} = G\delta \underline{x} \tag{6.77}$$

where

$$G = \frac{\partial \underline{g}}{\partial \underline{x}} \tag{6.78}$$

Equation (6.77) is a linear equation and has the solution

$$\delta \underline{x}_k = \Phi(T)\delta \underline{x}_{k-1} \tag{6.79}$$

where the state-transition matrix Φ satisfies the matrix differential equation

$$\dot{\Phi}(t) = G\Phi(t) \tag{6.80}$$

with $\Phi(0) = I$. As indicated by Eq. (6.79), the transition matrix of the linearized system measures the change in $\underline{x}_k$ per unit change in $\underline{x}_{k-1}$; therefore

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1})}{\partial \underline{x}_{k-1}} = F((k-1)T) = \int_0^T GF dt \tag{6.81}$$

with $F(0) = I$. Thus simultaneous integration of Eqs. (6.75) and (6.81) provides the information required by the estimator, and the analytic solution of the nonlinear plant differential equation is not required. An analytic comparison of Eq. (6.81) for the reactor equations is presented in Appendix E.

6.7 Performance of nuclear system state estimator

For the nuclear system nonlinear discrete-time Eqs. (4.75), (4.76) and (4.83), the performance index is defined:

$$J = [y_{k-1} - h(\underline{x}_{k-1})]^2 + [y_k - h(\underline{x}_k)]^2 \tag{6.82}$$

subject to

$$z_k = z_{k-1}\exp[\lambda\rho_{k-1}T/(1 - \rho_{k-1})] \tag{6.83}$$

$$\rho_k = \rho_{k-1} \tag{6.84}$$

The performance index is a minimum when

$$y_{k-1} = h(\underline{x}_{k-1}) = z_{k-1}/(1 - \rho_{k-1}) \tag{6.85}$$

$$y_k = h(\underline{x}_k) = z_k/(1 - \rho_k) \tag{6.86}$$

Using Eqs. (6.83), (6.84), (6.85), and (6.86) and two successive output samples, the solution for reactivity is

$$\hat{\rho}_{k-1} = \hat{\rho}_k = \frac{\ell n(y_k/y_{k-1})}{\lambda T + \ell n(y_k/y_{k-1})} \tag{6.87}$$

and for the delayed neutron precursor density

$$\hat{z}_{k-1} = (1 - \hat{\rho}_{k-1})y_{k-1} \tag{6.88}$$

The solution for $\hat{z}_k$ is obtained from Eq. (6.83) using Eqs. (6.87) and (6.88). Numerical values for the analytic solution of the estimator equations are obtained by using the Analytic Estimator Solutions computer program listed in Appendix G. The programmed value of y_0 is unity, and y_1 is calculated in response to a step change in reactivity occurring at t = (0+). Table 6.1 lists the analytic estimator solutions for different values of reactivity disturbances. These values are used to determine whether the estimator with iteration, programmed to solve Eqs. (6.70) and (6.71), generates the correct estimate in one sample after a disturbance.

The Finite Difference System with Estimator and Control computer program (listed in Appendix G), with the control loop opened by setting u = 0, generates samples of the output measurement by solving the plant finite difference Eqs. (6.83) and (6.84), the measurement Eq. (4.77),

TABLE 6.1

ANALYTIC ESTIMATOR SOLUTIONS

$\rho(0+)$	$\hat{\rho}_1$	$\hat{z}_1$	$\hat{n}_1$
0.25	0.55778428	0.65380766	1.4784811
0.20	0.49233880	0.68571204	1.3507277
0.10	0.31081242	0.79259985	1.1500494
-0.10	-0.66212729	1.4690358	0.88382870
-0.20	-3.0783149	3.2274608	0.79137116
-0.25	-11.471604	9.3774779	0.75190634

and the estimator Eqs. (6.70) and (6.71). Consecutive iterations of the estimator equations are performed until the performance index is equal to or less than a specified value, which can be expressed as

$$J \leq \varepsilon_1 \tag{6.89}$$

Thus by changing ε_1, the accuracy and number of iterations can be controlled.

The matrices F_{k-1} and H_k are obtained by differentiating Eqs. (6.83), (6.84), and (6.86), respectively, to obtain

$$F_{k-1} = \begin{bmatrix} \exp\left(\frac{\lambda T \rho_{k-1}}{1-\rho_{k-1}}\right) & \frac{\lambda T z_{k-1}}{(1-\rho_{k-1})^2 \exp\left(\frac{\lambda T \rho_{k-1}}{1-\rho_{k-1}}\right)} \\ 0 & 1 \end{bmatrix} \tag{6.90}$$

$$H_k = [1/(1 - \rho_k) \quad z_k/(1 - \rho_k)^2] \tag{6.91}$$

A worst-case analysis is used to investigate the performance of the estimator. Since a disturbance can occur anywhere within one sample interval, the worst case is when it occurs immediately after the measurement. The estimator is initialized by assuming the system to be in equilibrium up to t = 0. Thus

$$\underline{\alpha} = \underline{\hat{x}}_0 = \underline{x}(0-) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6.92}$$

$$H_\alpha = [1 \; 1] \tag{6.93}$$

and

$$\underline{x}(0+) = \begin{bmatrix} 1 \\ \rho(0+) \end{bmatrix} \tag{6.94}$$

For large reactivity disturbances, the first iteration produces an estimate of ρ_{k-1} which exceeds unity. If this happens, a discontinuity is crossed and the estimator is not able to converge. The computer program contains an arbitrary hard limit on ρ_{k-1} of 0.8. With this limit, the estimator produces correct estimates for step changes in reactivity up to +0.56\$. Similarly, a discontinuity exists at -0.27\$. Thus the useable range of the estimator for step disturbances is from -0.27\$ to +5.6\$.

Tables 6.2 and 6.3 show, respectively, estimator performance for $\hat{\rho}_k$, $\hat{z}_k$, and $\hat{n}_k$ in response to step reactivity disturbances of +0.1\$ and -0.1\$ with an iteration accuracy of $\epsilon_1 = 10^{-4}$. The number of iterations is indicated in column I, and the estimated values are given beneath the true values. For ρ = 0.1, the estimate is generated in four iterations and agrees up to the fifth decimal place with the values in Table 6.1. At the end of the second sample interval, the system state

TABLE 6.2

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	I
0	.10000000	1.00000005	1.11111114	
	.00000000*	1.00000005*	1.00000005*	
1	.10000000	1.03504454	1.15004947	
	.31082521*	.79259022*	1.15005689*	4
2	.10000000	1.07131714	1.19035235	
	.09907985*	1.07241802*	1.19035855*	3
3	.10000000	1.10886090	1.23206765	
	.09999066*	1.10867280*	1.23206808*	1
4	.10000000	1.14772035	1.27524482	
	.09999905*	1.14772158*	1.27524482*	1
5	.10000000	1.18794163	1.31993512	
	.09999996*	1.18794169*	1.31993512*	1
6	.10000000	1.22957243	1.36619157	
	.09999996*	1.22957249*	1.36619157*	1
7	.10000000	1.27266217	1.41406904	
	.09999996*	1.27266222*	1.41406904*	1
8	.10000000	1.31726195	1.46362438	
	.09999999*	1.31726200*	1.46362439*	1
9	.10000000	1.36342472	1.51491633	
	.09999993*	1.36342481*	1.51491633*	1
10	.10000000	1.41120523	1.56800579	
	.09999998*	1.41120525*	1.56800579*	1

*Estimate

TABLE 6.3

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, AND $\rho(0+) = -0.1$

k	ρ_k	z_k	n_k	I
0	-.10000000	1.00000005	.90909093	
	.00000000*	1.00000005*	1.00000005*	
1	-.10000000	.97221162	.88382875	
	-.66060630*	1.46791877*	.88396555*	3
2	-.10000000	.94519543	.85926857	
	-.09012711*	.93673362*	.85928843*	4
3	-.10000000	.91892997	.83539089	
	-.10006901*	.91900903*	.83541036*	1
4	-.10000000	.89339439	.81217672	
	-.10009105*	.89346834*	.81217672*	1
5	-.10000000	.86856840	.78960765	
	-.09999991*	.86856834*	.78960766*	1
6	-.10000000	.84443229	.76766572	
	-.09999999*	.84443226*	.76766571*	1
7	-.10000000	.82096686	.74633353	
	-.09999995*	.82096683*	.74633353*	1
8	-.10000000	.79815353	.72559412	
	-.09999991*	.79815348*	.72559413*	1
9	-.10000000	.77597413	.70543103	
	-.09999999*	.77597411*	.70543101*	1
10	-.10000000	.75441106	.68582824	
	-.09999995*	.75441103*	.68582824*	1

*Estimate

TABLE 6.4

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-6}$, AND $\rho(0+) = 0.25$

k	ρ_k	z_k	n_k	I
0	.25000001	1.00000005	1.33333338	
	.00000000*	1.00000005*	1.00000005*	
1	.25000001	1.10886100	1.47848131	
	.55778435*	.65380763*	1.47848132*	6
2	.25000001	1.22957266	1.63943018	
	.24997180*	1.22962016*	1.63943187*	4
3	.25000001	1.36342509	1.81790011	
	.24999822*	1.36342834*	1.81790009*	1
4	.25000001	1.51184884	2.01579849	
	.25000005*	1.51184879*	2.01579849*	1
5	.25000001	1.67643014	2.23524020	
	.25000005*	1.67643008*	2.23524020*	1
6	.25000001	1.85892791	2.47857056	
	.25000003*	1.85892788*	2.47857056*	1
7	.25000001	2.06129260	2.74839009	
	.25000003*	2.06129257*	2.74839009*	1
8	.25000001	2.28568686	3.04758244	
	.25000001*	2.28568683*	3.04758241*	1
9	.25000001	2.53450889	3.37934517	
	.25000004*	2.53450883*	3.37934520*	1
10	.25000001	2.81041793	3.74722387	
	.25000002*	2.81041790*	3.74722384*	1

*Estimate

TABLE 6.5

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-6}$, AND $\rho(0+) = -0.25$

k	ρ_k	z_k	n_k	I
0	-.25000001	1.00000005	.80000001	
	.00000000*	1.00000005*	1.00000005*	
1	-.25000001	.93988293	.75190635	
	-11.47042594*	9.37661416*	.75190811*	7
2	-.25000001	.88337989	.70670392	
	-.24271586*	.87823433*	.70670567*	7
3	-.25000001	.83027365	.66421894	
	-.24998595*	.83027136*	.66422455*	1
4	-.25000001	.78036003	.62428803	
	-.25004289*	.78038681*	.62428803*	1
5	-.25000001	.73344706	.58675765	
	-.24999987*	.73344699*	.58675765*	1
6	-.25000001	.68935435	.55148348	
	-.25000008*	.68935439*	.55148349*	1
7	-.25000001	.64791237	.51832990	
	-.24999993*	.64791234*	.51832991*	1
8	-.25000001	.60896177	.48716940	
	-.24999997*	.60896174*	.48716940*	1
9	-.25000001	.57235275	.45788219	
	-.24999992*	.57235272*	.45788220*	1
10	-.25000001	.53794457	.43035565	
	-.24999997*	.53794455*	.43035564*	1

*Estimate

TABLE 6.6

ESTIMATOR PERFORMANCE WITH INTEGRATED SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	Δt	I
0	.10000000	1.00000005	1.11111114		
	.00000000*	1.00000005*	1.00000005*		
1	.10000000	1.03504454	1.15004947		
	.31418966*	.78872010*	1.15005509*	1.00000	4
2	.10000000	1.07131714	1.19035235		
	.09946450*	1.07196009*	1.19035851*	1.00000	3
3	.10000000	1.10886090	1.23206765		
	.10033641*	1.10844578*	1.23206806*	1.00000	1
4	.10000000	1.14772035	1.27524482		
	.10034495*	1.14728047*	1.27524485*	1.00000	1
5	.10000000	1.18794163	1.31993512		
	.10034578*	1.18748521*	1.31993510*	1.00000	1
6	.10000000	1.22957243	1.36619157		
	.10034587*	1.22909993*	1.36619160*	1.00000	1
7	.10000000	1.27266217	1.41406904		
	.10034579*	1.27217317*	1.41406901*	1.00000	1
8	.10000000	1.31726195	1.46362438		
	.10034589*	1.31675571*	1.46362441*	1.00000	1
9	.10000000	1.36342472	1.51491633		
	.10034579*	1.36290086*	1.51491632*	1.00000	1
10	.10000000	1.41120523	1.56800579		
	.10034587*	1.41066292*	1.56800580*	1.00000	1

*Estimate

TABLE 6.7

ESTIMATOR PERFORMANCE WITH INTEGRATED SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	Δt	I
0	.10000000	1.00000005	1.11111114		
	.00000000*	1.00000005*	1.00000005*		
1	.10000000	1.03504454	1.15004947		
	.31165936*	.79163070*	1.15005660*	.25000	4
2	.10000000	1.07131714	1.19035235		
	.09925989*	1.07220364*	1.19035848*	.50000	3
3	.10000000	1.10886090	1.23206765		
	.10016345*	1.10865993*	1.23206809*	.50000	1
4	.10000000	1.14772035	1.27524482		
	.10017159*	1.14750156*	1.27524486*	.50000	1
5	.10000000	1.18794163	1.31993512		
	.10017248*	1.18771397*	1.31993513*	.50000	1
6	.10000000	1.22957243	1.36619157		
	.10017259*	1.22933565*	1.36619160*	.50000	1
7	.10000000	1.27266217	1.41406904		
	.10017251*	1.27241819*	1.41406901*	.50000	1
8	.10000000	1.31726195	1.46362438		
	.10017267*	1.31700924*	1.46362439*	.50000	1
9	.10000000	1.36342472	1.51491633		
	.10017254*	1.36316331*	1.51491630*	.50000	1
10	.10000000	1.41120523	1.56800579		
	.10017259*	1.41093461*	1.56800582*	.50000	1

*Estimate

is estimated to within 0.1%, requiring three iterations. Thereafter, a single iteration is used to track the system. The estimation sequence after k = 2, corresponds to an extended Kalman filter in which the previous estimate is used to evaluate the F and H matrices.

Tables 6.4 and 6.5 show, respectively, the estimator performance in response to reactivity disturbances of +0.25$ and -0.25$ with an iteration accuracy of $\varepsilon_1 = 10^{-6}$. The increase in iteration accuracy is required to obtain a good estimate for $\hat{\beta}_2$. For $\varepsilon_1 = 10^{-5}$, $\hat{\beta}_2 = -0.15607480$; and for $\varepsilon_1 = 10^{-4}$, $\hat{\beta}_2 = 0.08458834$. For $\varepsilon_1 = 10^{-4}$, $\hat{\beta}_3 = -0.25450338$ and one additional sample is required to obtain an accurate estimate of the system state. As indicated in Table 6.5, seven iterations are required for $\underline{\hat{x}}_1$ and $\underline{\hat{x}}_2$, and one iteration is used thereafter.

The estimates in Tables 6.2 to 6.5 are for a system described by finite-difference equations. The performance of an estimator which uses integration of the system equations is investigated by using the Differential System With Estimator and Control computer program (listed in Appendix G) with the feedback control loop opened by setting u = 0. The plant differential equations are given by Eqs. (4.32) and (4.33); and the variational equation used to calculate F_{k-1} is given by Eq. (4.60). The matrix differential equation to be integrated is

$$\begin{bmatrix} \dot{F}_{11} & \dot{F}_{12} \\ \dot{F}_{21} & \dot{F}_{22} \end{bmatrix} = \begin{bmatrix} \lambda\rho(t)/[1-\rho(t)] & z(t)/[1-\rho(t)]^2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \tag{6.95}$$

and after multiplication yields

$$\dot{F}_{11} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{11} + \frac{\lambda z(t)}{[1-\rho(t)]^2} F_{21}. \tag{6.96}$$

$$\dot{F}_{12} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{12} + \frac{\lambda z(t)}{[1-\rho(t)]^2} F_{22} \tag{6.97}$$

and

$$\dot{F}_{21} = \dot{F}_{22} = 0 \tag{6.98}$$

Solution of Eq. (6.98) requires that F_{21} = constant and F_{22} = constant, but the initial conditions require F(0) = I. Therefore, $F_{21} = 0$ and $F_{22} = 1$. After substitution of F_{21} and F_{22}, Eqs. (6.96) and (6.97) reduce to

$$\dot{F}_{11} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{11} \tag{6.99}$$

$$\dot{F}_{12} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{12} + \frac{\lambda z(t)}{[1-\rho(t)]^2} \tag{6.100}$$

The initial conditions are: $F_{11}(0) = 1$ and $F_{12}(0) = 0$. Simultaneous integration of Eqs. (4.32), (4.33), (6.99), and (6.100) yield the solutions for $\underline{x}_k$ and F_{k-1}.

The integration is performed numerically, therefore the accuracy of integration is dependent upon the step size. The Kutta-Merson method [126; 127, p. 24] given in Appendix F is used because of its one-step starting feature and error computation. The integration step size Δt is automatically adjusted to meet a specified accuracy requirement. The parameter ε_2 in the computer program, specifies the integration accuracy.

Tables 6.6 and 6.7 show, respectively, the estimator performance for a step change in reactivity of 0.1 with integration accuracies ε_2 of 10^{-2} and 10^{-3} and an iteration accuracy ε_1 of 10^{-4}. Comparison of Table 6.6 with Table 6.1 shows 1% accuracy of $\underline{x}_1$ and 0.5% accuracy for $\underline{x}_2$. For $\underline{x}_3$ and subsequent estimates, a steady error of approximately

0.34% is obtained for $\hat{\rho}$. The integration step size Δt, automatically selected by the integration subroutine, is shown to be 1 sec for each sample interval with the number of iterations remaining the same as in Table 6.2. Table 6.7 shows 0.3% accuracy for $\underline{x}_1$ with $\Delta t = 0.25$, and 0.74% accuracy for $\underline{x}_2$ with $\Delta t = 0.5$. For $\hat{x}_3$ and subsequent estimates, the steady error is 0.17% and $\Delta t = 0.5$. When $\Delta t = 0.25$, the equations of the integration subroutine are solved four times for each iteration, or 16 times for four iterations.

CHAPTER 7

COMBINED ESTIMATION AND CONTROL OF NUCLEAR SYSTEMS

7.1 Introduction

The problem of combined estimation and control has been investigated elsewhere [96, 97, 110, 125 and 128] with a resulting separation theorem. This theorem states that for linear systems subject to Gaussian noise with a quadratic cost function, the optimum stochastic controller is realized by cascading an optimal estimator with a deterministic optimum controller. The separation theorem does not apply to nonlinear systems with optimality guaranteed.

In Chapter 5, optimal control of a nuclear reactor was investigated using a control law which is a linear function of the state variables. The state variables: reactivity and delayed neutron precursor density, are not measureable. Therefore, in Chapter 6, an investigation was made of an optimal estimator which generates estimates of reactivity and delayed neutron precursor density from measurements of the prompt neutron density. In this chapter, the transient performance of the system is investigated with combined estimation and control.

7.2 Combined estimation and control

In Chapter 6, the estimator equations were derived with the assumption that the plant was not under control. With the plant under control, the linear prediction Eq. (6.32) is modified as follows:

$$\bar{\underline{x}}_k = \Phi_{k-1}\hat{\underline{x}}_{k-1} + Gu_{k-1} \tag{7.1}$$

and the linear filter Eq. (6.33) generates $\underline{\hat{x}}_k$ using $\underline{\bar{x}}_k$ of Eq. (7.1) and the y_k measurement. The control variable u_k is computed from

$$u_k = B\underline{\hat{x}}_k \tag{7.2}$$

and the predicted estimate of $\underline{\bar{x}}_{k+1}$ is obtained by using Eq. (7.1).

With control, the nonlinear plant Eq. (6.54) becomes

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}, u_{k-1}) \tag{7.3}$$

and Eq. (6.59) is written:

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1}, u_{k-1})}{\partial \underline{x}_{k-1}} \tag{7.4}$$

Equation (6.70) for the nonlinear filter remains unchanged, except that F_{k-1} is computed using Eq. (7.4), and the new form for Eq. (6.71) is

$$\underline{x}_k^{i+1} = \underline{f}(\underline{x}_{k-1}^{i}, u_{k-1}) + F_{k-1}(\underline{x}_{k-1}^{i+1} - \underline{x}_{k-1}^{i}) \tag{7.5}$$

At the end of the iteration sequence

$$\underline{\alpha}_k = \underline{x}_k^{i+1} \tag{7.6}$$

and the control variable u_k is computed from

$$u_k = B\underline{\alpha}_k \tag{7.7}$$

With control, the nonlinear plant Eq. (6.75) is

$$\dot{\underline{x}} = \underline{g}(\underline{x}, u) \tag{7.8}$$

and the value of $\underline{x}_k$ is obtained by integration:

$$\underline{x}_k = \underline{x}_{k-1} + \int_0^T \underline{g}(\underline{x}, u_{k-1})dt \tag{7.9}$$

The variational Eq. (6.77) remains unchanged, except that G defined by

Eq. (6.78), is replaced by

$$G = \frac{\partial \underline{g}(\underline{x}, u_{k-1})}{\partial \underline{x}} \tag{7.10}$$

7.3 Combined estimation and control with delay

The preceding calculation of the control variable assumed that a measurement is made at t = kT, the estimator equations are solved iteratively for a new estimate, the new control input is calculated, and the control is applied at t = kT. A more realistic control analysis should consider that a finite time is required to compute a new estimate and control input. The fastest sampling rate is determined by the time T required to execute the calculations outlined above.

The estimation equations remain valid, except that the control input must be delayed by one sample interval. Instead of using Eq. (7.7) to calculate the control at u_k, Eq. (7.3) with $\underline{\alpha}_{k-1}$ is used to predict the system state at t = kT:

$$\bar{\underline{x}}_k = \underline{f}(\underline{\alpha}_{k-1}, u_{k-1}) \tag{7.10}$$

Finally, the control input to be applied at t = kT is obtained using Eq. (7.10), with the result that

$$u_k = B\bar{\underline{x}}_k \tag{7.11}$$

If the calculations are completed in less than T seconds, u_k is stored, until t = kT, and then applied as an input after the measurement is made.

The new sequence is:

1. Obtain a measurement y_k.
2. Apply the previously calculated control input u_k.

3. Solve the estimator equations to obtain $\underline{\hat{x}}_k$.

4. Use the estimator output $\underline{\hat{x}}_k$ and control input u_k to predict the state of the plant at $t = (k+1)T$.

5. Use the predicted estimate $\underline{\bar{x}}_{k+1}$ to calculate a new control input u_{k+1}.

6. Store the control input u_{k+1} until the next measurement at $t = (k+1)T$.

7. Repeat the sequence.

If the total time to execute the above sequence is equal to the sampling period T, then the storage time is zero.

7.4 Nuclear control system performance

The performance of the control system, consisting of an estimator cascaded with the linear control law, is investigated with the plant described first by a difference equation and second by a differential equation.

The difference equation description of the plant is given by Eq. (4.78), and the measurement is given by Eq. (4.83).

The matrix F_{k-1} is obtained by differentiating Eq. (4.78) with the result that

$$F_{k-1} = \begin{bmatrix} \exp\left(\frac{\lambda}{u_{k-1}} \ln \frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}} T - \lambda T\right) & \frac{\lambda T z_{k-1}}{(1-\rho_{k-1})(1-\rho_{k-1}-u_{k-1})} \exp\left(\frac{\lambda}{u_{k-1}} \ln \frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}} T - \lambda T\right) \\ 0 & 1 \end{bmatrix} \tag{7.12}$$

and if $u_k = 0$, Eq. (6.90) is used. The matrix H_k is given by Eq. (6.91).

The nonlinear differential equation of the plant is given by Eq. (4.35), and the differential equations for F are given by Eqs. (6.99) and (6.100), and $F_{21} = 0$ and $F_{22} = 1$. In the finite-difference description, F_{k-1} is an explicit function of u_{k-1}, but in the differential description, F is not a direct function of u_{k-1}. The influence of control on F arises through the simultaneous integration of Eqs. (4.35), (6.99), and (6.100), as shown in Appendix E.

Figures 7.1 through 7.4 show the transient response for the system described by the finite-difference equations. These equations are solved by the Finite Difference System with Estimator and Control computer program. In Fig. 7.1, the response is for a step disturbance of $\rho = 0.1\$$ with no delay required for estimation and calculation of control effort. Since the disturbance occurs immediately after the measurement, the control for u_0 is zero. At the end of the first sample, the estimator generates an optimal estimate $\hat{n}_1$, which is the same as the value given in Table 6.2, and the control $u_1 = 0.173\$/sec$. After the second sample, the estimator generates the correct estimate of the system state, the control input is computed, and the neutron density deviation is driven to zero. For samples at t = 3 sec and greater, the neutron density deviation is zero, and the delayed neutron deviation and reactivity approach zero asymptotically.

The transient response plotted in Fig. 7.2 is obtained by calculating the control input using Eqs. (7.10) and (7.11). The estimate generated from the measurement made at t = 1 sec, is used with $u_1 = 0$ to obtain a predicted estimate $\bar{\underline{x}}_2$. This estimate is used to calculate u_2. The estimate generated from the measurement made at t = 2 sec gives the true state of the plant. The estimate $\hat{x}_2$ is used

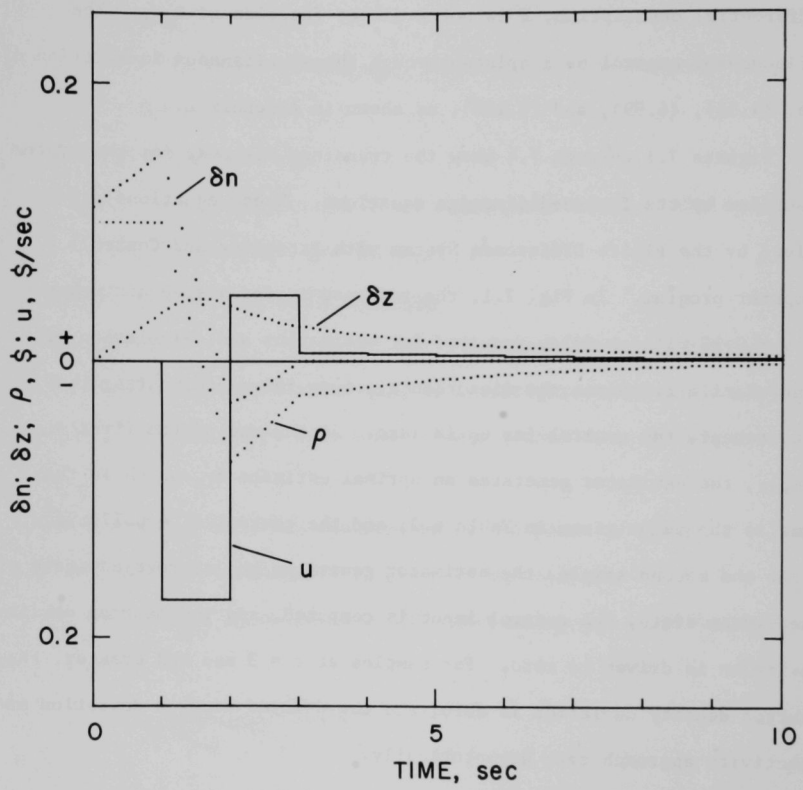

Fig. 7.1 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, without control delay.

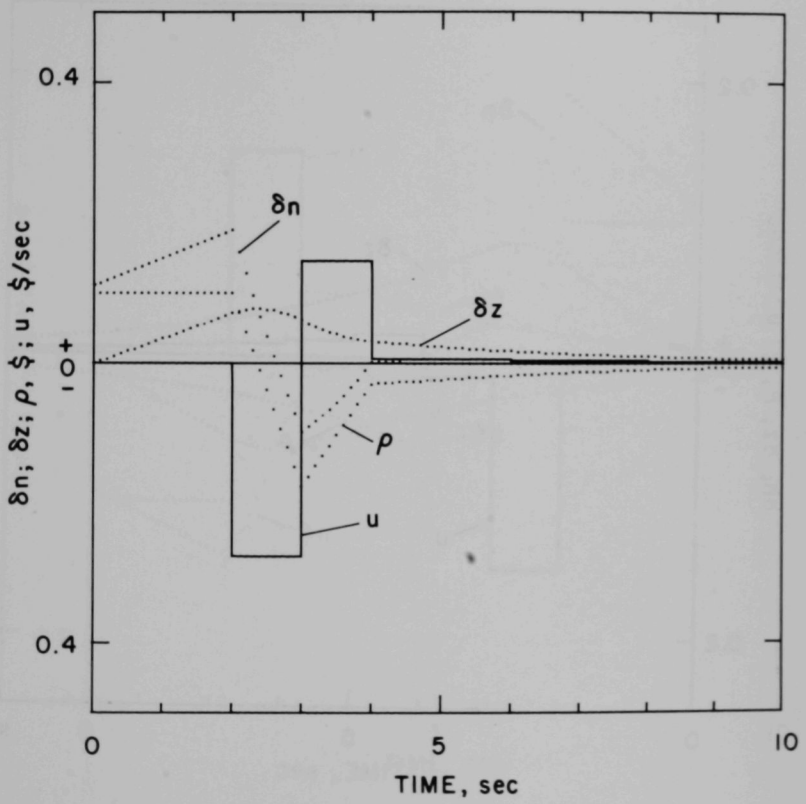

Fig. 7.2 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, with control delay.

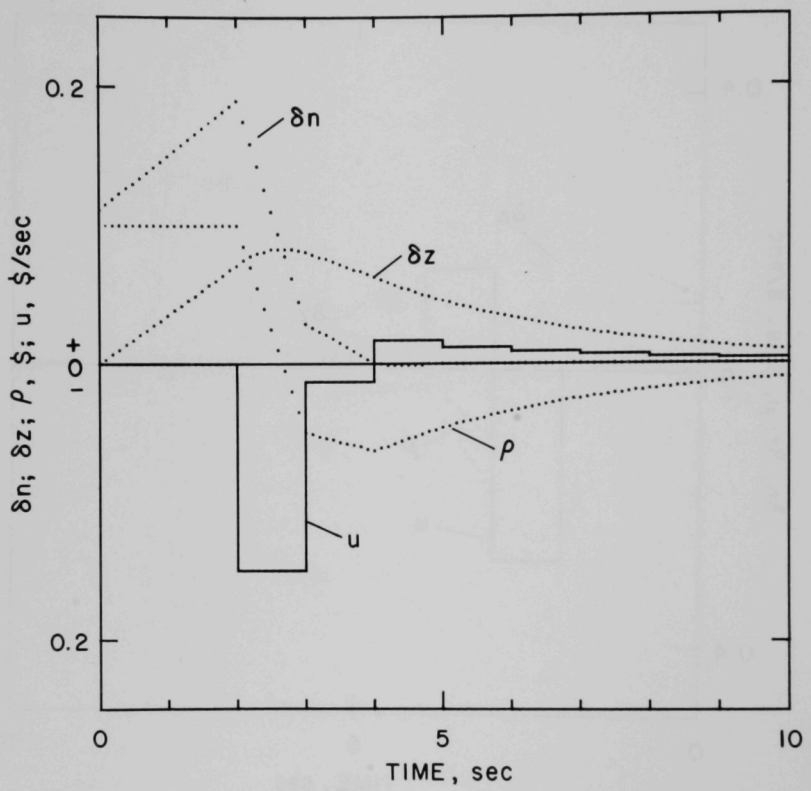

Fig. 7.3 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, with delayed and bounded control.

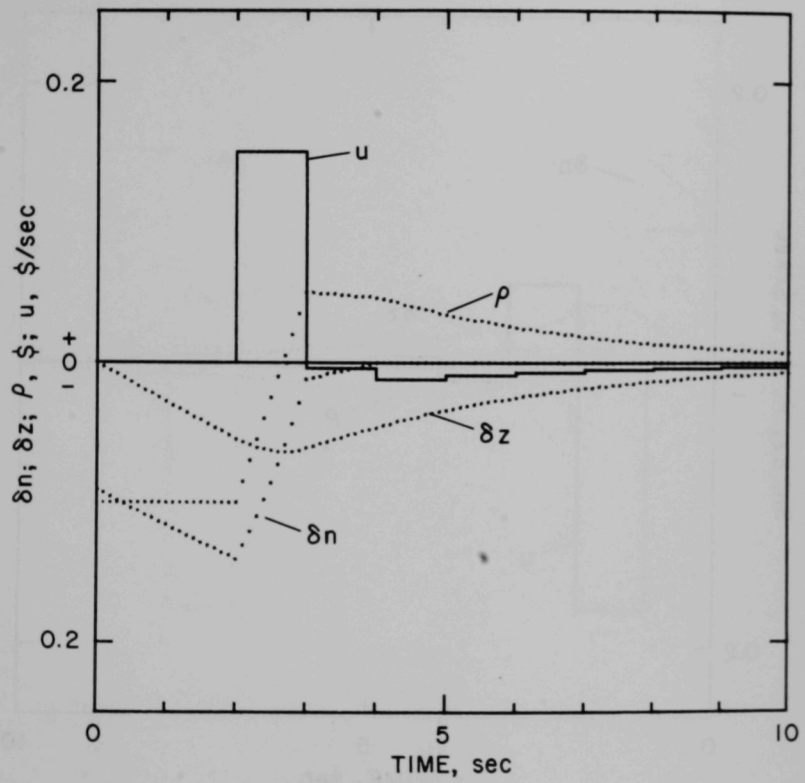

Fig. 7.4 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = -0.1$, with delayed and bounded control.

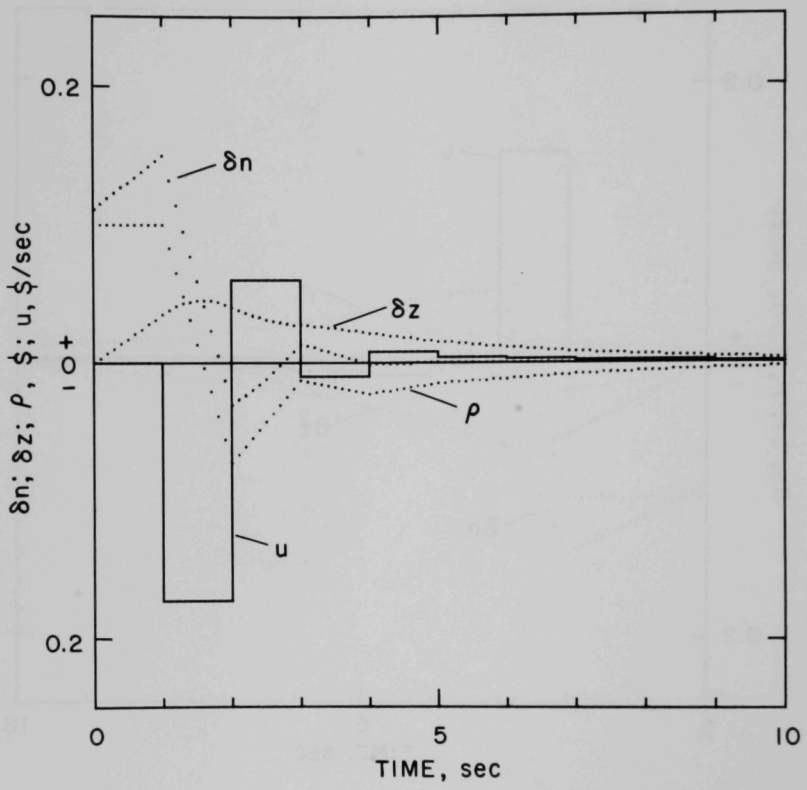

Fig. 7.5 **Transient** response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, without control delay.

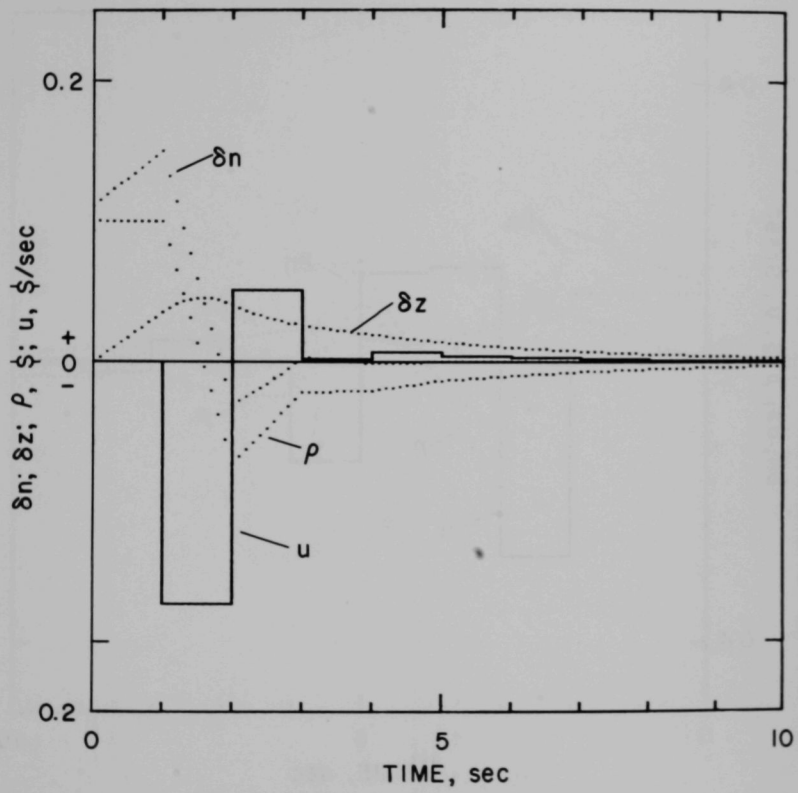

Fig. 7.6 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, without control delay.

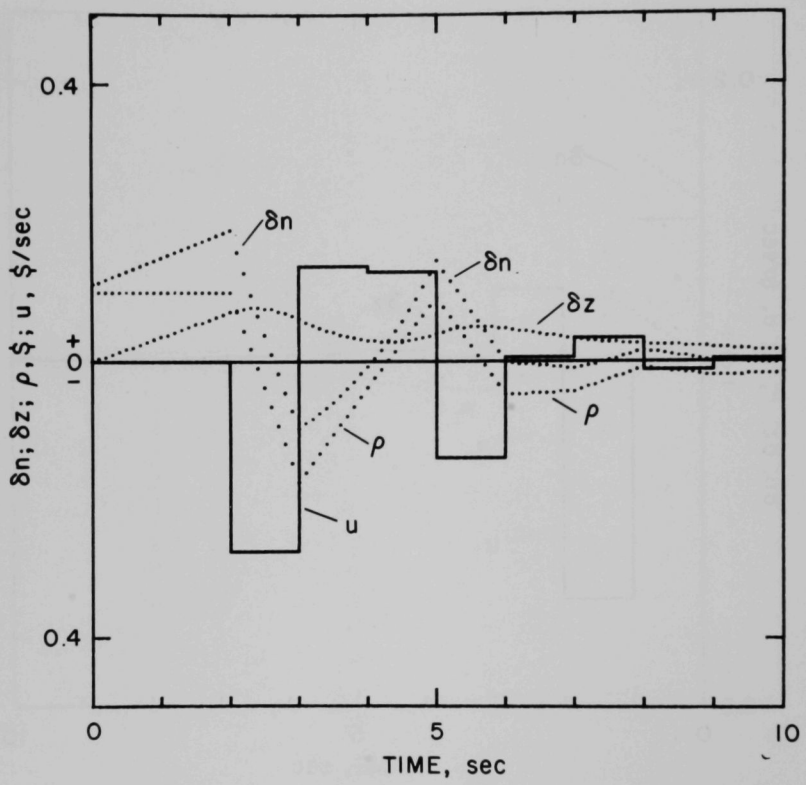

Fig. 7.7 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, with control delay.

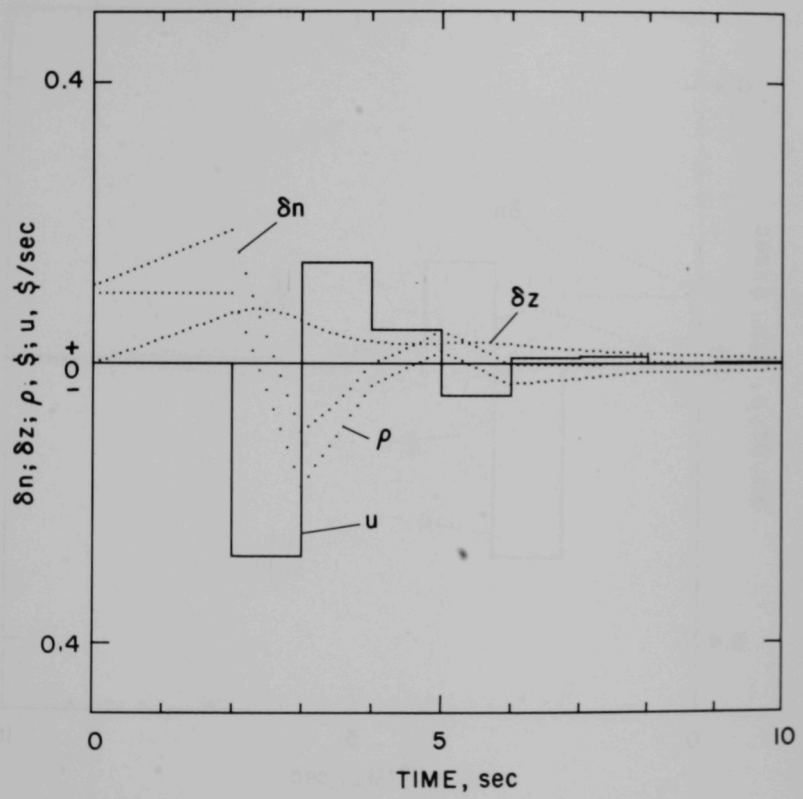

Fig. 7.8 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, with control delay.

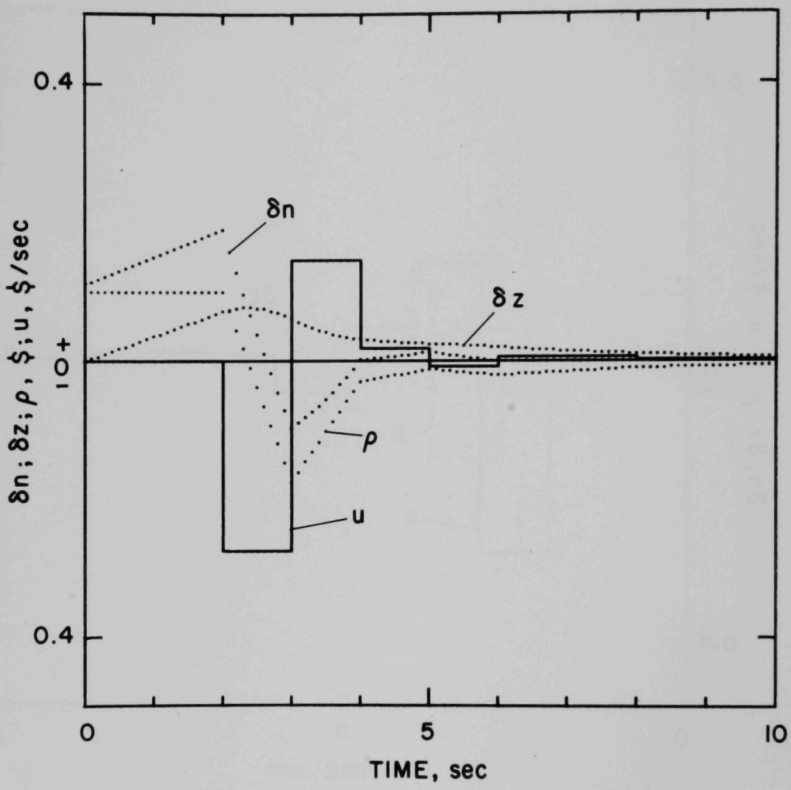

Fig. 7.9 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-5}$, $\varepsilon_2 = 10^{-4}$, and $\rho_0 = 0.1$, with control delay.

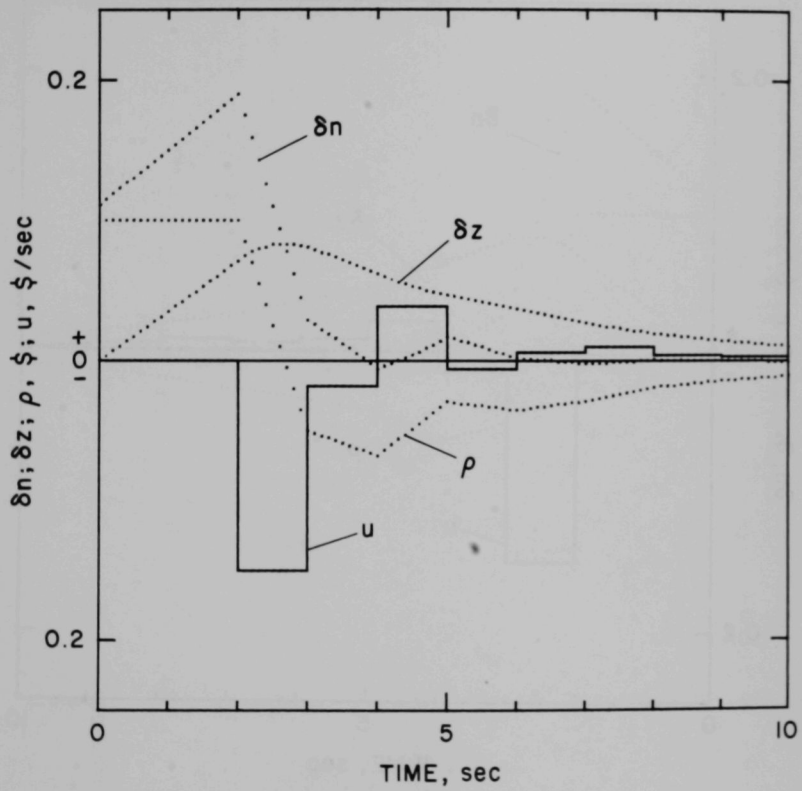

Fig. 7.10 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, with delayed and bounded control.

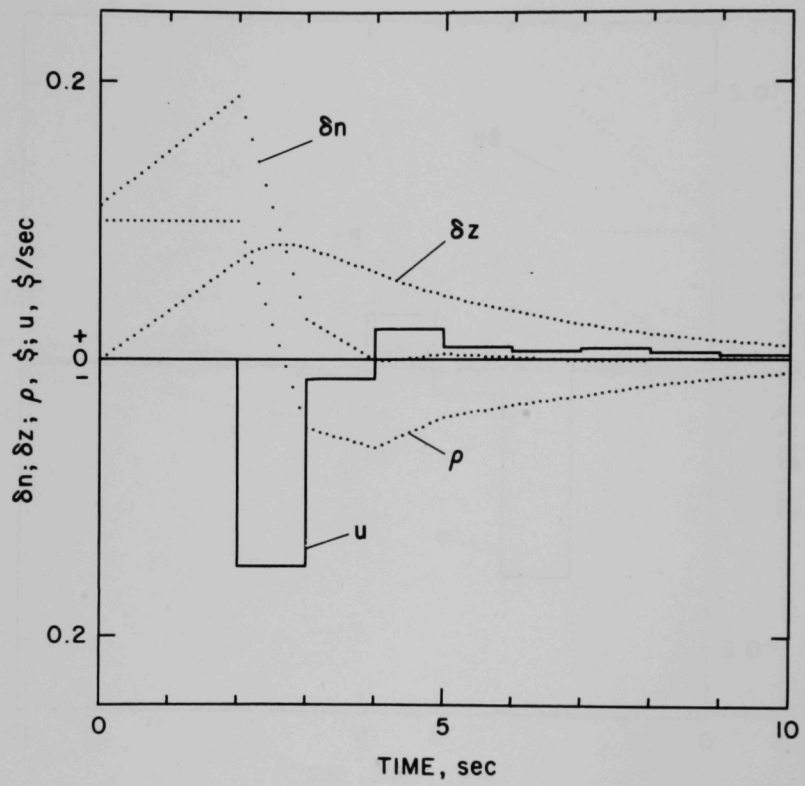

Fig. 7.11 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, with delayed and bounded control.

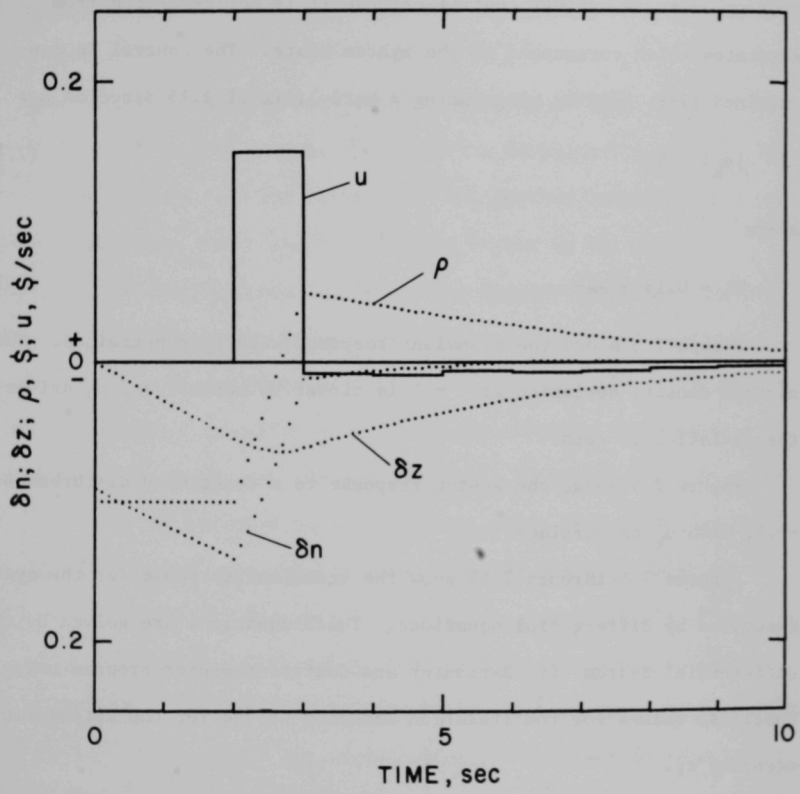

Fig. 7.12 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = -0.1$, with delayed and bounded control.

with u_2 to obtain a predicted estimate $\bar{\underline{x}}_3$. The control u_3, calculated using $\bar{\underline{x}}_3$, drives the neutron density deviation to zero, and the delayed neutron deviation and reactivity approach zero asymptotically.

The control input u_2 applied at t = 2 in Fig. 7.2 does not correspond to an optimal control because it is not generated from estimates which correspond to the system state. The control is constrained [129, 130] by programming a hard limit of 0.15 \$/sec on u_k:

$$|u_k| \leq UL \tag{7.13}$$

where

$$UL = 0.15 \text{ \$/sec} \tag{7.14}$$

Figure 7.3 shows the transient response with u_k constrained. The neutron density deviation at t = 3 is closer to normal, and u_3 drives the deviation to zero.

Figure 7.4 shows the system response to a reactivity disturbance of -0.1\$ with u_k constrained.

Figures 7.5 through 7.12 show the transient response for the system described by differential equations. These equations are solved by the Differential System with Estimator and Control computer program using specified values for the iteration accuracy ε_1 and for the integration accuracy ε_2.

The transient response to a step disturbance of ρ = 0.1\$ with no control delay is shown in Fig. 7.5 for $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-2}$. Comparison with Fig. 7.1 shows that the control u_2 does not drive the neutron density deviation to zero at t = 3 sec. This is due to an error in $\hat{x}_2$. However, the control u_3 drives the neutron density deviation to zero, and the reactivity and delayed neutron precursor density approach

zero asymptotically. In Fig. 7.6, where $\varepsilon_2 = 10^{-3}$, the estimate $\hat{x}_2$ is closer to the true state. This results in a u_2 which drives the neutron density to approximately zero. For the estimate $\underline{\hat{x}}_1$, $\Delta t = 0.125$ sec for the first iteration and $\Delta t = 0.25$ sec for the next three iterations. For the estimate $\underline{\hat{x}}_2$, $\Delta t = 0.25$ sec for two iterations. Thereafter, $\Delta t = 1$ sec.

The transient response with control delay is shown in Fig. 7.7 for $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-2}$. Here, the deviation in neutron density is 19% at t = 2 sec and $u_2 = -0.276$ \$/sec. The integration increment is 1 sec for all iterations, which results in large errors in the state estimates. The oscillations in the neutron density and reactivity are damped for this particular initial condition and set of parameters. Figure 7.8 shows the response with $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-3}$. The peak in the neutron density deviation at t = 5 sec is reduced, and greater damping is shown in the oscillatory behavior. In Fig. 7.9 for $\varepsilon_1 = 10^{-5}$ and $\varepsilon_2 = 10^{-4}$, the estimate $\underline{\hat{x}}_4$ results in u_5 which drives the neutron density deviation to zero. Except for a neutron density deviation of 1.26% at t = 5 sec, the response is similar to that plotted in Fig. 7.2. Four iterations are required for $\underline{\hat{x}}_1$ with an integration increment of $\Delta t = 0.0625$ sec. The three iterations for $\underline{\hat{x}}_2$ use integration increments of 0.0625, 0.125, and 0.25 sec, consecutively. Estimate $\underline{\hat{x}}_3$ is obtained in one iteration with $\Delta t = 0.0625$; $\underline{\hat{x}}_4$ is obtained in two iterations with $\Delta t = 0.125$ sec. The next two estimates, $\underline{\hat{x}}_5$ and $\underline{\hat{x}}_6$, are obtained in one iteration with $\Delta t = 0.5$ sec. Estimates for t = 7 sec and greater are obtained in one iteration with $\Delta t = 1$ sec. Thus $\underline{\hat{x}}_1$ requires the greatest number of calculations with 64 solutions of the integrator equations.

Figure 7.10 shows the transient response for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, with delay and a control bound of 0.15 \$/sec. The integration increment is 0.5 sec for the first iteration and is 1 sec thereafter. In comparison with Fig. 7.3, the neutron density deviation has an error of 1.7% at $t = 5$ sec.

In Fig. 7.11 where $\varepsilon_2 = 10^{-3}$, the neutron density deviation at $t = 5$ sec is 0.4%. The first iteration requires a $\Delta t = 0.125$ sec, and the next three iterations are with $\Delta t = 0.25$ sec to obtain $\hat{x}_1$. For $\hat{x}_2$, three iterations are required with $\Delta t = 0.25$ sec, whereas, one iteration is required with $\Delta t = 1$ sec for succeeding estimates. Figure 7.12 shows the system response for a reactivity disturbance of -0.1\$.

CHAPTER 8

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

8.1 Summary

The six-group point-model kinetics equations for a nuclear reactor were normalized, solved with a step change in reactivity, and compared to the transient response obtained using a prompt-jump approximation. This demonstrated that for the control system investigation, it is satisfactory to use the prompt-jump approximation with a resulting reduction in order of the system. A further approximation was introduced by using a single group of delayed neutrons. The decay constant for the one-group model was selected by making a comparison with the transient response of the six-group model at 1 sec. The rate of change of reactivity was chosen as a control input by neglecting the control rod motor time constant.

State-space concepts were introduced and vector matrix notation was used to express: the six-group point model kinetics equation, the normalized six-group kinetics equation, the six-group prompt-jump model, the one-group kinetics equation, and the one-group prompt-jump model. A first-order Taylor series expansion was used to linearize the one-group prompt-jump equation. The one-group prompt-jump equation and the linearized equations were solved with a discrete-time input.

An optimal stationary feedback control law was used to minimize a quadratic performance index for a discrete-time system. A

performance index was defined which consisted of the sum of the squares of the neutron density deviation. This index was augmented to include terms in reactivity and control. For selected values of the weighting coefficients, the stationary feedback matrix was calculated using an iterative digital computer program. System transient behavior was plotted to demonstrate the influence of the weighting coefficients. For the performance index as defined, the neutron density deviation is driven to zero in one sample interval after a step disturbance in reactivity. The control law assumes that all state variables are available, but the specific variables reactivity and delayed neutron precursor density cannot be measured.

Kalman's filter was derived for a linear deterministic system by a matrix inversion lemma and by minimization of a least-squares cost function. The resulting filter equations showed the relationship of the optimal filter gain to the state transition and measurement matrices. A nonlinear estimator was derived by minimizing a least-squares performance index and iteration was used to solve the resulting nonlinear equations. The filter derivations were based on the assumption that the system was described by finite-difference equations. Therefore, the plant and variational equations were integrated to obtain the necessary numerical values required by the estimator.

An algebraic solution of the reactor equations was derived to obtain the estimated system state after a step disturbance in reactivity. This solution was compared to the solution obtained by iteration to measure the performance of the nonlinear estimator. A digital computer program was used to solve the estimator equations and iterations were performed automatically until the estimator performance index was

reduced to a specified value. The performance of the estimator for a nuclear system described by finite-difference equations was investigated with different iteration accuracies. Because of its one-step starting feature and error estimation, the Kutta-Merson algorithm was used to integrate the plant and variational equations. The error estimate was used to automatically adjust the integration step size to meet a specified accuracy requirement. The performance of the estimator using integration was investigated as a function of iteration accuracy and integration accuracy.

Control of a nuclear reactor was investigated by cascading the optimal estimator with the optimal controller. After a reactivity disturbance, the optimal estimator requires two samples to estimate the true state of the plant. After the second sample, the optimal controller drives the neutron density deviation to zero in one sample. If it is assumed that one sample interval is required to perform the estimation and control calculations, then the delayed neutron deviation is driven to zero in one sample after the third measurement is made. A constraint on the control variable was introduced to reduce the magnitude of the control input applied after the second estimate is made. The performance of the cascaded control system with an estimator using integration was investigated as a function of iteration accuracy and integration accuracy. With a small integration step size, system performance with integration is equal to that of the system described by finite-difference equations. The penalty for increased accuracy is an increase in computation time.

8.2 Conclusions

The optimal control law derived for a discrete-time linear system with a quadratic cost function demonstrated that a deviation in neutron density could be reduced to zero in one sample interval. The stationary feedback control law for the reactor was derived by linearizing the reactor equations around the desired nominal values. The plotted responses (Figs. 5.1 - 5.12) are idealistic because the optimal control requires knowledge of the reactivity and delayed neutron precursor density at each sampling instant. From a process standpoint, this is a physical impossibility, because these variables are not measureable and therefore must be estimated.

The nonlinear estimator using iteration works very well for a system described by nonlinear plant and measurement difference equations. If integration is used to estimate the state of a system described by a nonlinear differential equation, the integration step size must be reduced to maintain estimation accuracy; as a consequence, the computation time is increased. For higher-order systems, the combination of iteration and sequential integration can easily result in an estimation time exceeding one second. Integration of a set of simultaneous equations can be more profitably assigned to an analog computer with a factor of ten applied to the problem time scale. Thus, an integration over one sample interval in problem time can be obtained in one-tenth of a sample interval in real time. The number of equations to be integrated will not change the integration time, since all equations are integrated simultaneously. Thus, the nonlinear estimator becomes a hybrid system, with a digital computer solving the estimator difference

equations and an analog computer solving the system differential equations.

The cascade combination of an estimator and controller results in a control system whose performance is no longer equal to that of a system without an estimator. Whereas, the linearized reactor equations result in a linear stationary control law which controls the nonlinear system satisfactorily under the assumption that all state variables are measureable, the performance of the cascaded system demonstrates that the estimates generated for the nonlinear system result in a large control input at the first sampling instant after a disturbance. Inclusion of computation time delay results in further degraded performance. A bound on the control variable can be used to limit the control inputs until the estimator establishes the true state of the system. If an integrator is included as part of the nonlinear estimator, the integration step size must be reduced to even smaller values when a control input is present.

The computer programs used to solve the estimator equations and to compute the control input are not compiled for minimum time execution; therefore, no conclusions can be made as to real-time control capability.

8.3 Recommendations for future research

A hybrid computer system should be used to establish feasibility of real-time control. An analog computer should be used to simulate the reactor system, and a digital computer should be used for the estimation and control calculations. The reactor equations should be expanded to include six groups of delayed neutrons. Use of the six-group model will

encourage inclusion of an integrator in the estimator, because an analytic description by finite-difference equations will be difficult.

Starting with the one-group model, the regulator problem should be investigated with noise added to the plant and measurement equations. The stochastic system should be expanded to include the six-group model.

The deterministic and stochastic one-group and six-group models should be used to investigate control of demand changes in reactor power level from source range to power operation, with and without reactivity feedback.

At very low power levels, a nuclear reaction is a multiplicative Poisson process. Optimal estimation theory should be applied to the design of a reactivity meter.

The methods of estimation and control applied to the kinetics equations should be expanded to include the primary system, the secondary system, and the turbine-generator system, with automatic start-up, operation, and shutdown.

Optimal control theory should be used to establish ultimate system performance without regard to cost. Since total optimization of the control of a nuclear plant includes the performance of the controller and its cost, an investigation should be made to determine whether a significant savings in equipment cost is possible by accepting slightly less than optimal performance.

APPENDIX A

VECTOR-MATRIX DIFFERENTIAL EQUATIONS

The homogeneous differential equation for a linear time-invariant system is given in vector-matrix form by

$$\dot{\underline{x}}(t) = A\underline{x}(t), \quad \underline{x}(t_0) = \underline{x}_0 \tag{A.1}$$

The solution to Eq. (A.1) is

$$\underline{x}(t) = \Phi(t - t_0)\underline{x}(t_0) \tag{A.2}$$

where the state transition matrix is defined by

$$\Phi(t - t_0) = \exp[A(t - t_0)] \tag{A.3}$$

The matrix exp(At) is defined by the infinite series

$$\exp(At) = I + At + A^2t^2/2! + A^3t^3/3! + \cdots \tag{A.4}$$

Substitution of Eq. (A.2) into Eq. (A.1) yields

$$\dot{\Phi}(t - t_0) = A\Phi(t - t_0) \tag{A.5}$$

Use of Eq. (A.4) in Eq. (A.5) verifies that Eq. (A.2) is a solution of Eq. (A.1). Note that when $t = t_0$,

$$\Phi(0) = I \tag{A.6}$$

and the boundary conditions of Eq. (A.2) are satisfied.

The state transition matrix $\Phi(t)$ can be calculated by using Eq. (A.4)

$$\Phi(t) = I + At + A^2t^2/2! + A^3t^3/3! + \cdots \tag{A.7}$$

or by taking the Laplace transform of both sides of Eq. (A.1) to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) \tag{A.8}$$

Rearrangement of Eq. (A.8) leads to

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) \tag{A.9}$$

which alternately can be written as

$$\underline{X}(s) = \Phi(s)\underline{x}(0) \tag{A.10}$$

where $\Phi(s)$, the resolvent of matrix A, is given by

$$\Phi(s) = [sI - A]^{-1} \tag{A.11}$$

The state transition matrix $\Phi(t)$ is obtained by taking the inverse Laplace transform of both sides of Eq. (A.11) which can be expressed:

$$\Phi(t) = \mathcal{L}^{-1}[sI - A]^{-1} \tag{A.12}$$

The solution to the nonhomogeneous equation

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) \tag{A.13}$$

is obtained by first taking the Laplace transform of both sides to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) + BU(s) \tag{A.14}$$

rearranging

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) + [sI - A]^{-1}BU(s) \tag{A.15}$$

and then taking the inverse Laplace transform of both sides with the result that

$$\underline{x}(t) = \Phi(t)\underline{x}(0) + \int_0^t \Phi(t - \tau)Bu(\tau)d\tau \tag{A.16}$$

where the convolution theorem is used to obtain the integral term.

If the initial time is given as t_0 instead of zero, then

$$\underline{x}(t) = \Phi(t - t_0)\underline{x}(t_0) + \int_{t_0}^{t} \Phi(t - \tau)Bu(\tau)d\tau \qquad (A.17)$$

For a discrete-time input u_k where

$$u(t) = u_k \qquad kT < t \leq (k+1)T \qquad (A.18)$$

Eq. (A.17) is written

$$\underline{x}(t) = \phi(t - t_k)\underline{x}_k + u_k \int_{t_k}^{t} \phi(t - \tau)Bd\tau \qquad (A.19)$$

or

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{0}^{t-t_k} \Phi(\tau)Bd\tau \qquad (A.20)$$

The integral term of Eq. (A.20) can be evaluated by integrating Eq. (A.7) from zero to T:

$$\int_0^T \Phi(\tau)d\tau = IT + AT^2/2 + A^2T^3/3! + \cdots \qquad (A.21)$$

Multiplication by A of both sides of Eq. (A.21) yields

$$A\int_0^T \Phi(\tau)d\tau = AT + A^2T^2/2 + A^3T^3/3! + \cdots \qquad (A.22)$$

The unit matrix can be added to both sides of Eq. (A.22) as follows:

$$I + A\int_0^T \Phi(\tau)d\tau = I + AT + A^2T^2/2 + A^3T^3/3! + \cdots \qquad (A.23)$$

but the right hand side of Eq. (A.23) is $\Phi(T)$. Therefore,

$$I + A\int_0^T \Phi(\tau)d\tau = \Phi(T) \tag{A.24}$$

which can be rewritten

$$\int_0^{t-t_k} \Phi(\tau)d\tau = A^{-1}[\Phi(t - t_k) - I] \tag{A.25}$$

if A^{-1} exists. Substitution of Eq. (A.25) into Eq. (A.20) yields

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + A^{-1}[\Phi(t - t_k) - I]Bu_k \tag{A.26}$$

and at $t = (k+1)T$

$$\underline{x}_{k+1} = \Phi(T)\underline{x}_k + A^{-1}[\Phi(T) - I]Bu_k \tag{A.27}$$

The homogeneous matrix differential equation of a time-varying linear system is

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t), \quad \underline{x}(t_0) = x_0 \tag{A.28}$$

Any solution of Eq. (A.28) is given by

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) \tag{A.29}$$

This is verified by substituting Eq. (A.29) into Eq. (A.28) with the result that

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0) \tag{A.30}$$

and

$$\begin{aligned}\dot{\underline{x}}(t) &= \frac{d}{dt}[\Phi(t, t_0)\underline{x}(t_0)] \\ &= A(t)\Phi(t, t_0)\underline{x}(t_0) \\ &= A(t)\underline{x}(t)\end{aligned} \tag{A.31}$$

Also

$$\Phi(t_0, t_0) = I \tag{A.32}$$

and the boundary conditions are satisfied. Integration of Eq. (A.28) yields

$$\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} A(\tau)\underline{x}(\tau)d\tau \tag{A.33}$$

which can be solved by repeated substitution of the right side into the integral for $\underline{x}$. The first substitution yields

$$\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} A(\tau)[\underline{x}(t_0) + \int_{t_0}^{\tau} A(\nu)\underline{x}(\nu)d\nu]d\tau \tag{A.34}$$

Define the operator

$$Q(\ ) = \int_{t_0}^{t} (\ )d\tau \tag{A.35}$$

which leads to the following series as a solution of Eq. (A.22):

$$\underline{x}(t) = [I + Q(A) + Q(AQ(A)) + Q(AQ(AQ(A))) + \cdots]\underline{x}(t_0) \tag{A.36}$$

Comparison of Eq. (A.36) with Eq. (A.29) shows that the state transition matrix for a time-varying system is given by:

$$\Phi(t, t_0) = I + Q(A) + Q(AQ(A)) + Q(AQ(AQ(A))) + \cdots \tag{A.37}$$

If A is constant matrix, then

$$\Phi(t, t_0) = I + A(t - t_0) + A^2(t - t_0)^2/2! + A^3(t - t_0)^3/3! + \cdots \tag{A.38}$$

which is the same as Eq. (A.7) with the argument replaced by $t - t_0$.

Assume that the solution of the nonhomogeneous differential equation

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) + B(t)u(t), \quad \underline{x}(t_0) = \underline{x}_0 \tag{A.39}$$

is given by

$$\underline{x}(t) = \Phi(t, t_0)\underline{y}(t) \tag{A.40}$$

Then

$$\dot{\underline{x}}(t) = \Phi(t, t_0)\dot{\underline{y}}(t) + \dot{\Phi}(t, t_0)\underline{y}(t) \tag{A.41}$$

and Eq. (A.30) is substituted into Eq. (A.41) to eliminate $\dot{\Phi}$. Thus

$$\dot{\underline{x}}(t) = \Phi(t, t_0)\dot{\underline{y}}(t) + A(t)\Phi(t, t_0)\underline{y}(t) \tag{A.42}$$

Substitution of Eq. (A.39) into Eq. (A.38) results in

$$\dot{\underline{x}}(t) = A(t)\Phi(t, t_0)\underline{y}(t) + B(t)u(t) \tag{A.43}$$

which on comparison with Eq. (A.42) results in

$$\Phi(t, t_0)\dot{\underline{y}}(t) = B(t)u(t) \tag{A.44}$$

and y(t) is obtained by integration. Thus

$$\underline{y}(t) = \underline{y}(t_0) + \int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.45}$$

At $t = t_0$, Eqs. (A.32) and (A.40) result in

$$\underline{y}(t_0) = \underline{x}(t_0) \tag{A.46}$$

Equation (A.40) is solved for $\underline{y}(t)$ and substituted with Eq. (A.46) into Eq. (A.45) to yield

$$\Phi^{-1}(t, t_0)\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.47}$$

The solution for $\underline{x}(t)$ is

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) + \Phi(t, t_0)\int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.48}$$

Using the properties of the state transition matrix

$$\Phi^{-1}(\tau, t_0) = \Phi(t_0, \tau) \tag{A.49}$$

and

$$\Phi(t, t_0)\Phi(t_0, \tau) = \Phi(t, \tau) \tag{A.50}$$

Eq. (A.48) can be written

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) + \int_{t_0}^{t} \Phi(t,\tau)B(\tau)u(\tau)d\tau \tag{A.51}$$

For the discrete-time input defined by Eq. (A.18), Eq. (A.51) becomes

$$\underline{x}(t_{k+1}) = \Phi(t_{k+1}, t_k)\underline{x}(t_k) + u_k \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)B(\tau)d\tau \tag{A.52}$$

which can be written

$$\underline{x}_{k+1} = \Phi(k+1, k)\underline{x}_k + u_k \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)B(\tau)d\tau \tag{A.53}$$

APPENDIX B

OPTIMAL CONTROL LAW FOR A DISCRETE-TIME LINEAR SYSTEM WITH A QUADRATIC PERFORMANCE INDEX

For a discrete-time linear system described by

$$\underline{x}_{k+1} = \Phi \underline{x}_k + G u_k \tag{B.1}$$

and a quadratic performance index of the form

$$I_N = \sum_{k=1}^{N} (\underline{x}_k^T Q \underline{x}_k + c u_{k-1}^2) \tag{B.2}$$

the optimal control law can be found by the method of dynamic programming.

There is a sequence: $u_0, u_1, u_2, \cdots, u_{N-1}$ which will make I_N a minimum. Let the minimum value of I_N be denoted by

$$f_N[\underline{x}(0)] = \min_{\substack{u_0 \\ u_1 \\ \cdots \\ u_{N-1}}} \sum_{k=1}^{N} [\underline{x}_k^T Q \underline{x}_k + c u_{k-1}^2] \tag{B.3}$$

For the last N-j stages of an N-stage process

$$f_{N-j}[\underline{x}(j)] = \min_{\substack{u_j \\ u_{j+1} \\ \cdots \\ u_{N-1}}} \sum_{k=j+1}^{N} [\underline{x}_k^T Q \underline{x}_k + c u_{k-1}^2] \tag{B.4}$$

The principle of optimality [132, p. 57] may be used to interpret the selection of $u_0, u_1, u_2, \cdots, u_{N-1}$ as a sequence of decision processes. The principle of optimality states: "An optimal policy has the

property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Then, by the principle of optimality, Eq. (B.4) reduces to

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T Q \underline{x}_{j+1} + cu_j^2 + f_{N-(j+1)}(\underline{x}_{j+1})] \tag{B.5}$$

Starting with j = 0,

$$f_N(\underline{x}_0) = \min_{u_0} [\underline{x}_1^T Q \underline{x}_1 + cu_0^2 + f_{N-1}(\underline{x}_1)] \tag{B.6}$$

$$f_{N-1}(\underline{x}_1) = \min_{u_1} [\underline{x}_2^T Q \underline{x}_2 + cu_1^2 + f_{N-2}(\underline{x}_2)] \tag{B.7}$$

$$\vdots$$

$$f_1(\underline{x}_{N-1}) = \min_{u_{N-1}} [\underline{x}_N^T Q \underline{x}_N + cu_{N-1}^2 + f_0(x_N)] \tag{B.8}$$

Define

$$f_0(\underline{x}_N) = 0 \tag{B.9}$$

Since the functional f is quadratic in $\underline{x}$, both f_{N-j} and $f_{N-(j+1)}$ can be expressed in quadratic forms. Let

$$f_{N-j}(\underline{x}_j) = \underline{x}_j^T P_{N-j} \underline{x}_j \tag{B.10}$$

and

$$f_{N-(j+1)}(\underline{x}_{j+1}) = \underline{x}_{j+1}^T P_{N-(j+1)} \underline{x}_{j+1} \tag{B.11}$$

where the P matrices are n×n and symmetrical.

On substitution of Eq. (B.11) into Eq. (B.5)

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T Q \underline{x}_{j+1} + cu_j^2 + \underline{x}_{j+1}^T P_{N-(j+1)} \underline{x}_{j+1}] \tag{B.12}$$

Define

$$S_{N-(j+1)} = Q + P_{N-(j+1)} \tag{B.13}$$

Then

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T S_{N-(j+1)} x_{j+1} + cu_j^2] \tag{B.14}$$

but x_{j+1} is a function of u_j. Then, after substitution of Eq. (B.1), Eq. (B.14) becomes

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [(\Phi\underline{x}_j + Gu_j)^T S_{N-(j+1)} (\Phi\underline{x}_j + Gu_j) + cu_j^2] \tag{B.15}$$

The minimum of Eq. (B.15) may be found by taking the derivative with respect to u_j and equating the result to zero. Thus

$$2[\Phi\underline{x}_j + Gu_j]^T S_{N-(j+1)} G + 2cu_j = 0 \tag{B.16}$$

which can be expanded to give

$$\underline{x}_j^T \Phi S_{N-(j+1)} G + G^T S_{N-(j+1)} Gu_j + cu_j = 0 \tag{B.17}$$

Taking the transpose of Eq. (B.17) and solving for u_j results in

$$u_j = - \frac{G^T S_{N-(j+1)} \Phi}{G^T S_{N-(j+1)} G + c} \underline{x}_j \tag{B.18}$$

which may be expressed in linear form by

$$u_j = B_{N-j} \underline{x}_j \tag{B.19}$$

where

$$B_{N-j} = - \frac{G^T S_{N-(j+1)} \Phi}{G^T S_{N-(j+1)} G + c} \tag{B.20}$$

or

$$B_{N-j} = \frac{G^T[Q + P_{N-(j+1)}]\Phi}{G^T[Q + P_{N-(j+1)}]G + c} \tag{B.21}$$

The recurrence relationship for the P matrices is obtained by substituting Eqs. (B.10), (B.13), and (B.19) into Eq. (B.15) to obtain

$$\underline{x}_j^T P_{N-j}\underline{x}_j = \underline{x}_j^T(\Phi + GB_{N-j})^T(Q + P_{N-(j+1)})(\Phi + GB_{N-j})\underline{x}_j + c\underline{x}_j^T B_{N-j}^T B_{N-j}\underline{x}_j \tag{B.22}$$

Comparing both sides of Eq. (B.22) leads to

$$P_{N-j} = (\Phi + GB_{N-j})^T(Q + P_{N-(j+1)})(\Phi + GB_{N-j}) + cB_{N-j}^T B_{N-j} \tag{B.23}$$

Equations (B.21) and (B.23) give the desired recurrence relationship for the B and P matrices. Starting with $j = N-1$, and $P = 0$, the sequence is: $B_1, P_1, B_2, P_2, \cdots, P_{N-1}, B_N$.

When $N \to \infty$ in Eq. (B.2), the control process becomes an infinite stage process, and the feedback control law given by Eq. (B.19) becomes time invariant.

APPENDIX C

SERIES EXPANSION OF DISCRETE-TIME REACTOR EQUATION

Integration of the reactor kinetics equations results in the following discrete-time solution for the normalized delayed neutron precursor density:

$$z_{k+1} = z_k \exp\left[\frac{\lambda}{u_k}\ell n \frac{1 - \rho_k}{1 - \rho_k - u_k T} - \lambda T\right] \tag{C.1}$$

which is unsatisfactory for numerical computation as $u_k \to 0$.

Equation (C.1) may be expanded in a Taylor series by defining

$$f(u_k) = \ell n \frac{1 - \rho_k}{1 - \rho_k - u_k T} \tag{C.2}$$

Then

$$f'(u_k) = \frac{T}{1 - \rho_k - u_k T} \tag{C.3}$$

$$f''(u_k) = \frac{T^2}{(1 - \rho_k - u_k T)^2} \tag{C.4}$$

$$f'''(u_k) = \frac{2T^3}{(1 - \rho_k - u_k T)^3} \tag{C.5}$$

and

$$f^n(u_k) = \frac{(n - 1)!\, T^n}{(1 - \rho_k - u_k T)^n} \tag{C.6}$$

The Taylor series expansion for Eq. (C.2) is

$$f(u_k) = 0 + \frac{T}{1-\rho_k} u_k + \frac{T^2}{2(1-\rho_k)^2} u_k^2 + \cdots + \frac{T^n}{n(1-\rho_k)^n} u_k^n + \cdots \quad (C.7)$$

Substitution of Eq. (C.7) into Eq. (C.1) results in

$$z_{k+1} = z_k \exp\left[\frac{\lambda T}{1-\rho_k} + \frac{\lambda T^2}{2(1-\rho_k)^2} u_k + \cdots + \frac{\lambda T^n}{n(1-\rho_k)^n} u_k^{n-1} + \cdots - \lambda T\right] \quad (C.8)$$

The first and last terms inside of the bracket may be combined with the result that

$$z_{k+1} = z_k \exp\left[\frac{\lambda T \rho_k}{1-\rho_k} + \frac{\lambda T^2}{2(1-\rho_k)^2} u_k + \cdots + \frac{\lambda T^n}{n(1-\rho_k)^n} u_k^{n-1} + \cdots\right] \quad (C.9)$$

Let

$$x = u_k T/(1 - \rho_k) \quad (C.10)$$

Then Eq. (C.9) may be expressed as follows

$$z_{k+1} = z_k \exp\left[\frac{\lambda T}{1-\rho_k} \left(\rho_k + \frac{x}{2} + \frac{x^2}{3} + \cdots + \frac{x^{n-1}}{n} + \cdots\right)\right] \quad (C.11)$$

When $u_k = 0$, $x = 0$, and Eq. (C.11) reduces to Eq. (4.75).

APPENDIX D

PROOF THAT ONE ERROR TERM OF OPTIMAL ESTIMATOR IS ZERO

For the nonlinear estimator with iteration, the estimate $\underline{x}_{k-1}^{i+1}$ is given by

$$\underline{x}_{k-1}^{i+1} = \underline{x}_{k-1}^{i} + [\underline{H}_{\alpha-\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]^{-1}\{F_{k-1}^{T}H_{k}^{T}[y_{k} - h(\underline{x}_{k}^{i}) + H_{k}(\underline{x}_{k}^{i} - \underline{f}(\underline{x}_{k-1}^{i}))] + \underline{H}_{\alpha-\alpha}^{T}\underline{H}(\underline{\alpha} - \underline{x}_{k-1}^{i})\} \tag{D.1}$$

It can be demonstrated that

$$\underline{H}_{\alpha}(\underline{\alpha} - \underline{x}_{k-1}^{i}) = 0 \tag{D.2}$$

throughout the iteration sequence by first rearranging Eq. (D.1) and multiplying both sides to obtain

$$[\underline{H}_{\alpha-\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]\underline{x}_{k-1}^{i+1} = F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}\underline{x}_{k-1}^{i} + F_{k-1}^{T}H_{k}^{T}[y_{k} - h(\underline{x}_{k}^{i}) + H_{k}(\underline{x}_{k}^{i} - f(\underline{x}_{k-1}^{i}))] + \underline{H}_{\alpha-\alpha}^{T}\underline{H}_{\alpha} \tag{D.3}$$

Then, if the term $F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}\underline{\alpha}$ is added and subtracted to the right side

$$[\underline{H}_{\alpha-\alpha}^{T}\underline{H} + F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}]\underline{x}_{k-1}^{i+1} = F_{k-1}^{T}H_{k}^{T}H_{k}F_{k-1}\underline{\alpha} + \underline{H}_{\alpha-\alpha}^{T}\underline{H}_{\alpha} + F_{k-1}^{T}H_{k}^{T}[y_{k} - h(\underline{x}_{k}^{i}) + H_{k}\underline{x}_{k}^{i} - H_{k}f(\underline{x}_{k-1}^{i}) - H_{k}F_{k-1}\underline{\alpha} + H_{k}F_{k-1}\underline{x}_{k-1}^{i}] \tag{D.4}$$

If β^{i} is defined

$$\underline{\beta}^i = f(\underline{x}_{k-1}^i) + F_{k-1}[\underline{\alpha} - \underline{x}_{k-1}^i] \tag{D.5}$$

substituted into Eq. (D.4)

$$[\underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1}]\underline{x}_{k-1}^{i+1} = [\underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1}]\underline{\alpha} + F_{k-1}^T H_k^T[y_k - h(\underline{x}_k^i) + H_k(\underline{x}_k^i - \underline{\beta}^i)] \tag{D.6}$$

and both sides are multiplied

$$\underline{x}_{k-1}^{i+1} = \underline{\alpha} + [\underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1}]^{-1}F_{k-1}^T H_k^T[y_k - h(\underline{x}_k^i) + H_k(\underline{x}_k^i - \underline{\beta}^i)] \tag{D.7}$$

If the term $\underline{\alpha}$ is subtracted from both sides of Eq. (D.7) and the resulting equation is multiplied by $\underline{H}_{\alpha}$

$$\underline{H}_{\alpha}(\underline{x}_{k-1}^{i+1} - \underline{\alpha}) = \underline{H}_{\alpha}[\underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1}]^{-1}F_{k-1}^T H_k^T[y_k - h(\underline{x}_k^i) + H_k(\underline{x}_k^i - \underline{\beta}^i)] \tag{D.8}$$

If the left side of Eq. (D.8) is equal to a zero column vector and the error terms in the bracket on the right side are not zero, then

$$\underline{H}_{\alpha}[\underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1}]^{-1}F_{k-1}^T H_k^T = \underline{0} \tag{D.9}$$

The equality of Eq. (D.9) can be demonstrated by using the matrix inversion

$$\underline{x}_{k-1} = \begin{bmatrix} \underline{H}_{\alpha} \\ \hline H_k F_{k-1} \end{bmatrix}^{-1}\begin{bmatrix} \underline{y}_{\alpha} \\ \hline y_k \end{bmatrix} \tag{D.10}$$

or

$$\underline{x}_{k-1} = M^{-1}[\underline{H}_{\alpha}^T \,\vdots\, F_{k-1}^T H_k^T]\begin{bmatrix} \underline{y}_{\alpha} \\ \hline y_k \end{bmatrix} \tag{D.11}$$

where

$$M = \underline{H}_{\alpha}^T\underline{H}_{\alpha} + F_{k-1}^T H_k^T H_k F_{k-1} \tag{D.12}$$

Multiplying both sides of Eq. (D.11) by the composite H matrix yields

$$\begin{bmatrix} \underline{H}_\alpha \\ \hline F_{k-1}H_k \end{bmatrix} \underline{x}_{k-1} = \begin{bmatrix} \underline{H}_\alpha \\ \hline F_{k-1}H_k \end{bmatrix} M^{-1} [\underline{H}_\alpha^T \mid F_{k-1}^T H_k^T] \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \tag{D.13}$$

The left side is equal to the measurement vector, therefore Eq. (D.13) can be written

$$\begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} = \left[\begin{array}{c|c} \underline{H}_\alpha M^{-1} \underline{H}_\alpha^T & \underline{H}_\alpha M^{-1} F_{k-1}^T H_k^T \\ \hline F_{k-1}H_k M^{-1} \underline{H}_\alpha^T & F_{k-1}H_k M^{-1} F_{k-1}^T H_k^T \end{array}\right] \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \tag{D.14}$$

Since $\underline{y}_\alpha$ and y_k are independent, the partioned matrix of Eq. (D.14) is an n×n unit matrix, and

$$\underline{H}_\alpha M^{-1} \underline{H}_\alpha^T = I \qquad (n-1 \times n-1) \tag{D.15}$$

$$\underline{H}_\alpha M^{-1} F_{k-1}^T H_k^T = \underline{0} \qquad (n-1 \times 1) \tag{D.16}$$

$$F_{k-1}H_k M^{-1} \underline{H}_\alpha^T = \underline{0} \qquad (1 \times n-1) \tag{D.17}$$

$$F_{k-1}H_k M^{-1} F_{k-1}^T H_k^T = 1 \tag{D.18}$$

Equation (D.9) is verified by Eq. (D.16).

Since the iteration sequence is started with $\underline{x}_{k-1}^1 = \underline{\alpha}$, the first error correction term contributed by $\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^1)$ is zero, and all subsequent values are zero. Therefore, the term $\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i)$ may be omitted from the estimator.

APPENDIX E

DIFFERENCE SOLUTIONS BY INTEGRATION

Although numerical integration is used in the digital computer calculation of the reactor state estimates, the reactor equations can be integrated analytically to demonstrate that F_{k-1} obtained by integration is equal to F_{k-1} obtained by differentiation of the plant difference equation. The following equations are integrated simultaneously from zero to t:

$$\dot{z}(t) = \lambda z(t)\rho(t)/[1 - \rho(t)] \qquad z(0) = z_{k-1} \qquad \text{(E.1)}$$

$$\dot{\rho}(t) = u_{k-1} \qquad \rho(0) = \rho_{k-1} \qquad \text{(E.2)}$$

$$\dot{\phi}_{11}(t) = \lambda\rho(t)\phi_{11}(t)/[1 - \rho(t)] \qquad \phi_{11}(0) = 1 \qquad \text{(E.3)}$$

$$\dot{\phi}_{12}(t) = \lambda\rho(t)\phi_{11}(t)/[1 - \rho(t)] + \lambda z(t)/[1 - \rho(t)]^2 \qquad \phi_{12}(0) = 0 \qquad \text{(E.4)}$$

First, Eq. (E.2) is integrated to obtain

$$\rho(t) = \rho_{k-1} + u_{k-1}t \qquad \text{(E.5)}$$

which is substituted into Eq. (E.1), yielding

$$z(t) = z_{k-1}\exp\left[\frac{\lambda}{u_{k-1}}\,\ell n\left(\frac{1 - \rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \qquad \text{(E.6)}$$

Next Eq. (E.5) is substituted into Eq. (E.3) and integrated

$$\phi_{11}(t) = \exp\left[\frac{\lambda}{u_{k-1}} \ell n \left(\frac{1 - \rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \tag{E.7}$$

Finally, Eqs. (E.5), (E.6), and (E.7) are substituted into Eq. (E.4) and integrated, with the result

$$\phi_{12}(t) = \frac{\lambda t z_{k-1}}{(1-\rho_{k-1})(1-\rho_{k-1}-u_{k-1}t)} \exp\left[\frac{\lambda}{u_{k-1}} \ell n \left(\frac{1 - \rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \tag{E.8}$$

At t = T, Eqs. (E.7) and (E.8) agree with matrix Eq. (7.12), which is obtained by differentiating the plant finite-difference equations.

APPENDIX F

KUTTA-MERSON INTEGRATION ALGORITHM

Merson [126] proposed an integration method which does not require a special starting feature and which can be used with automatic interval adjustment. The Kutta-Merson process uses the equations

$$y_1 = y_0 + \frac{1}{3}hf(x_0, y_0) \tag{F.1}$$

$$y_2 = y_0 + \frac{1}{6}hf(x_0, y_0) + \frac{1}{6}hf(x_0 + \frac{1}{3}h, y_1) \tag{F.2}$$

$$y_3 = y_0 + \frac{1}{8}hf(x_0, y_0) + \frac{3}{8}hf(x_0 + \frac{1}{3}h, y_2) \tag{F.3}$$

$$y_4 = y_0 + \frac{1}{2}hf(x_0, y_0) - \frac{3}{2}hf(x_0 + \frac{1}{3}h, y_2) + 2hf(x_0 + \frac{1}{2}h, y_3) \tag{F.4}$$

$$y_5 = y_0 + \frac{1}{6}hf(x_0, y_0) + \frac{2}{3}hf(x_0 + \frac{1}{2}h, y_3) + \frac{1}{6}hf(x_0 + h, y_4) \tag{F.5}$$

Merson showed that the error in y_4 is $-h^5y^{(v)}/120$, and in y_5 is $-h^5y^{(v)}/720$; and that a good estimate of the error in the computed y_5 is $0.2(y_4 - y_5)$.

Automatic interval adjustment is accomplished by specifying the integration accuracy ε_2 and adjusting h. If

$$|0.2(y_4 - y_5)| > \varepsilon_2 \tag{F.6}$$

h is halved. If

$$64\ |0.2(y_4 - y_5)| < \varepsilon_2 \tag{F.7}$$

then h is doubled.

The advantage of the Kutta-Merson method is that it facilitates rapid interval selection for exploratory calculations requiring specified accuracy; however it does require additional computation time in comparison to other methods.

APPENDIX G

DIGITAL COMPUTER PROGRAMS

1. ROOTS OF PROMPT JUMP EQUATION

```
      DIMENSION S(7), B(7), A(7) , D(7),ST(7)
    1 FORMAT (1H1)
    2 FORMAT (F12.8)
    3 FORMAT (9X, 1HB, 16X, 1HS)
    4 FORMAT (E14.8, 4X, E14.8)
    5 FORMAT (1H )
      A(1) = 0.038
      A(2) = 0.213
      A(3) = 0.188
      A(4) = 0.407
      A(5) = 0.128
      A(6) = 0.026
      D(1) = 0.0127
      D(2) = 0.0317
      D(3) = 0.115
      D(4) = 0.311
      D(5) = 1.40
      D(6) = 3.87
      S(1) = 0.01
      S(2) = -0.014
      S(3)= -0.065
      S(4) = -0.19
      S(5) = -1.25
      S(6) = -3.75
      S(7) = -65000.0
      ALPHA = 65000.0
      X = 1.0
      RHO = 0.1
      ERR = 1.0E-7
   10 DO 15 I = 1,7
      ST(I) = 0.0
   11 SUM 1 = 0.0
      SUM 2 = 0.0
      DO 12 J = 1,6
      SUM1 = SUM1 + A(J) *(S(I)/(S(I)+D(J)))**2
      SUM2 = SUM2 + A(J)*D(J)/(S(I)+D(J))**2
   12 CONTINUE
      S(I) = (RHO - SUM1)/(SUM2 + X/ALPHA)
      DIFF = S(I) - ST(I)
      ST(I) = S(I)
      WRITE TYPE 2, S(I)
      IF (DIFF) 16, 17, 17
   16 DIFF = - DIFF
   17 IF(DIFF - ERR) 13, 13, 11
   13 SUM 3 = 0.0
      DO 14 K = 1,6
      SUM3 = SUM3 + A(K)/(S(I) + D(K))
   14 CONTINUE
```

```
      B(I) =(SUM3 + X/ALPHA)/(SUM2 + X/ALPHA)
      WRITE TYPE 5
   15 CONTINUE
      PRINT 1
      PRINT 3
      PRINT 4, (B(I), S(I), I=1,7)
      PUNCH 4, (B(I), S(I), I=1,7)
      END
```

2. REACTOR RESPONSE TO STEP DELTA K

```
      DIMENSION S(7), B(7)
    1 FORMAT (1H1)
    2 FORMAT (2F16.8)
    3 FORMAT (6X, 4HTIME, 12X, 4HFLUX, 12X, 6HFLUX 1,/)
      READ 2, (B(I), S(I), I=1,7)
      PRINT 1
      PRINT 3
      DELT = 0.0001
      DO 11 N = 1,101
      T = (N - 1) * DELT
      FLUX = 0.0
      DO 10 I = 1,7
   10 FLUX = FLUX + B(I) * EXPF(S(I)*T)
      PRINT 2, T, FLUX
   11 CONTINUE
      END
```

3. CALCULATION OF FEEDBACK MATRIX

```
      DIMENSION PHI(7,7),H(7),Q(7,7),P(7,7),S(7,7),HTSPHI(7),
     1B(7),PSI(7,7),PSIPSI(7,7),BTB(7,7)
   10 FORMAT (F16.8)
   21 FORMAT (4F16.8)
   11 FORMAT (I1)
      READ 11, N
      READ 10,  ALAMBD,T
   20 READ 10, A, C
   12 FORMAT (1H1//40X51HCALCULATION OF FEEDBACK MATRIX WITH
     1CONTROL PENALTY)
   13 FORMAT (1H0,9X,6HLAMBDA,20X,1HT,23X,1HA,23X,1HC,23X,1HN)
      PRINT 12
      PRINT 13
   14 FORMAT (1H ,4(4X,F16.8,4X),11X,I1)
      PRINT 14, ALAMBD,T,A,C,N
   15 FORMAT (1H0,24X,3HPHI,32X,1HH,33X,1HQ)
      PRINT 15
   16 FORMAT (1H0,10X,2F16.8,10X,F16.8,10X,2F16.8)
   17 FORMAT (1H0,14X,1HS,29X,1HB,28X,3HPSI,28X,1HP)
   18 FORMAT (1H0,1X,3(2F14.8,2X),2F14.8)
   19 FORMAT (1H0,1X,2F14.8,32X,2F14.8,2X,2F14.8)
      PHI(1,1) = 1.
      PHI(1,2) = T * ALAMBD
      PHI(2,1) = 0.
      PHI(2,2) = 1.
      H(1) = 0.5 * T ** 2 * ALAMBD
      H(2) = T
      Q(1,1) = 1.
      Q(1,2) = 1.
      Q(2,1) = 1.
      Q(2,2) = 1. + A
      PRINT 16, PHI(1,1),PHI(1,2),H(1),Q(1,1),Q(1,2)
      PRINT 16, PHI(2,1),PHI(2,2),H(2),Q(2,1),Q(2,2)
      PRINT 17
      DO 100 I = 1,N
      DO 100 J = 1,N
  100 P(I,J) = 0.
      B1TEMP = 0.0
      B2TEMP = 0.0
   99 DO 101 I = 1,N
      DO 101 J = 1,N
  101 S(I,J) = Q(I,J) + P(I,J)
      HTSH = 0.
      DO 102 I = 1,N
      DO 102 J = 1,N
  102 HTSH = HTSH + H(I) * S(I,J) * H(J)
      DEN = HTSH + C
      DO 105 K = 1,N
```

```
      HTSPHI (K) = 0.
      DO 104 I =1,N
      DO 104 J = 1,N
  104 HTSPHI (K) = HTSPHI (K) + H(I) * S(I,J) * PHI(J,K)
  105 B(K) = -HTSPHI(K)/DEN
      DO 106 I = 1,N
      DO 106 J = 1,N
  106 PSI(I,J) = PHI(I,J) + H(I) * B(J)
      DO 107 I = 1,N
      DO 107 J = 1,N
      PSIPSI(I,J) = 0.
      DO 107 K = 1,N
      DO 107 L = 1,N
  107 PSIPSI(I,J) = PSIPSI(I,J) + PSI(K,I) * S(K,L) * PSI(L,J)
      DO 108 I = 1,N
      DO 108 J = 1,N
  108 BTB(I,J) = B(I) * B(J)
      DO 109 I = 1,N
      DO 109 J = 1,N
  109 P(I,J) = PSIPSI(I,J) + C * BTB(I,J)
      DIFF1 = B1TEMP - B(1)
      IF (DIFF1) 30,31,31
   30 DIFF1 = -DIFF1
   31 IF(DIFF1 - 0.00000001) 33,33,32
   32 B1TEMP = B(1)
      GO TO 99
   33 DIFF2 = B2TEMP - B(2)
      IF (DIFF2) 34,35,35
   34 DIFF2 = -DIFF2
   35 IF(DIFF2 - 0.00000001) 37,37,36
   36 B2TEMP = B(2)
      GO TO 99
   37 PRINT 18, S(1,1),S(1,2),B(1),B(2),PSI(1,1),PSI(1,2),
     1P(1,1),P(1,2)
      PRINT 19, S(2,1),S(2,2),PSI(2,1),PSI(2,2),P(2,1),P(2,2)
      PUNCH 21, B(1), B(2), A, C
      GO TO 20
  111 END
```

4. CALCULATION OF TRANSIENT RESPONSE

```
10 FORMAT (1H1,28X,18HTRANSIENT RESPONSE)
11 FORMAT (1H0,20X,2HB1,27X,2HB2)
12 FORMAT (1H ,14X,F14.8,15X,F14.8,59X,F4.0)
13 FORMAT (F14.8)
14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX,
  116X,2HPI/)
15 FORMAT (1H ,I4,4(3X,F16.8))
16 FORMAT (I5,4F14.8)
17 FORMAT (I5,F14.8)
18 FORMAT (I5)
19 FORMAT (4F16.8)
25 FORMAT (84X,F16.8)
   READ 18, M
   READ 13, T
   READ 13, ALAMBD
   READ 13, RHO0
   READ 13, Z0
20 READ 13, RUNNO
   READ 19, B1, B2, A,C
   PRINT 10
   PRINT 11
   PRINT 12, B1, B2, RUNNO
   PRINT 14
   N = 0
   PI = 0.
   FLUX = Z0/(1.0 - RHO0)
   U = B1 * (Z0 - 1.0) + B2 * RHO0
   RHO = RHO0
   DELTAN = FLUX - 1.0
   DELTAZ = Z0 - 1.0
   PRINT 15, N, U, RHO, DELTAZ, DELTAN
   PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   ZK = Z0
   RHOK = RHO0
21 DO 22 K = 1,10
   X = U*K*T/(1.0-RHOK)
   SER = (ALAMBD*K*T/(1.0-RHOK))*(RHOK+X*(1.0/2.0+X*(1.0/3.0
  1+X*(1.0/4.0+X*(1.0/5.0+X*(1.0/6.0+X*(1.0/7.0+X*(1.0/8.0+
  2X*(1.0/9.0+X/10.0)))))))))
   Z = ZK*EXPF(SER)
   RHO = RHOK + U*K*T
   FLUX = Z/(1.0-RHO)
   DELTAN = FLUX - 1.0
   DELTAZ = Z - 1.0
   N = N+1
   PRINT 15, N, U, RHO, DELTAZ, DELTAN
   PUNCH 16, N, U, RHO, DELTAZ, DELTAN
22 CONTINUE
```

```
      ZK = Z
      RHOK = RHO
      PI = PI + (FLUX - 1.0)**2 + A*RHO**2 + C*U**2
      U = B1*(ZK-1.0)+B2*RHOK
      PUNCH 17,N,U
      PRINT 25, PI
      IF (N-M) 21,23,23
   23 IF(SENSE SWITCH 1) 24,20
   24 PAUSE 1
      GO TO 20
      END
```

5. PLOT PROGRAM FOR TRANSIENT RESPONSE

```
      DIMENSION N(111),U(111),RHO(111),Z(111),FLUX(111),A(111)
    1 FORMAT (F6.0)
    2 FORMAT (5F16.8)
    3 FORMAT (I5,4F14.8)
    6 FORMAT (1H0, 5F16.8,4X,F6.0)
      SN = 20.0
      S = 0.2
      SU = S
      SRHO = S
      SZ = S
      SFLUX = S
    9 READ 1, RUN NO
   12 READ 3, (N(I),U(I),RHO(I),Z(I),FLUX(I),I=1,111)
      IRUNNO = RUN NO
      DIGIT1 = IRUNNO/10
      DIGIT2 = RUN NO - DIGIT1 * 10.
      DO 27 J = 1,3
      X = PLOTF (2.0,2.0,1)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF (0.0,0.02,3)
      X = PLOTF (DIGIT1,0.0,4)
      X = PLOTF (10.0,0.0,3)
      X = PLOTF(0.0,0.0,2)
      X = PLOTF(DIGIT2,0.0,4)
      X = PLOTF (0.0,-11.0,3)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF (1.0,1.0,1)
      X = PLOTF (0.0,2.5,3)
      X = PLOTF (0.0,-2.5,4)
      X = PLOTF (0.0,2.0,3)
      X = PLOTF (0.12,2.0,4)
      X = PLOTF (0.0,2.0,3)
      X = PLOTF (0.0,0.0,3)
      X = PLOTF (5.0,0.0,4)
      X = PLOTF (0.0,0.0,3)
      X = PLOTF (0.0,-2.0,3)
      X = PLOTF (0.12,-2.0,4)
      X = PLOTF (0.0,-2.0,3)
      X = PLOTF (0.0,-2.5,3)
      DO 34 I = 1,9
      T = 0.5 * I
      X = PLOTF (T,-2.5,4)
      IF(I - 5) 31,32,31
   31 X = PLOTF (T,-2.44,4)
      GO TO 33
   32 X = PLOTF (T,-2.38,4)
   33 X = PLOTF (T,-2.5,3)
   34 CONTINUE
```

```
      X = PLOTF (5.0,-2.5,4)
      X = PLOTF (5.0,-2.0,4)
      X = PLOTF (4.88,-2.0,4)
      X = PLOTF (5.0,-2.0,3)
      X = PLOTF (5.0,2.0,4)
      X = PLOTF (4.88,2.0,4)
      X = PLOTF (5.0,2.0,3)
      X = PLOTF (5.0,2.5,4)
      DO 38 I = 1,9
      T = 5.0 - 0.5 * I
      X = PLOTF (T,2.5,4)
      IF(I - 5) 35,36,35
   35 X = PLOTF (T,2.44,4)
      GO TO 37
   36 X = PLOTF (T,2.38,4)
   37 X = PLOTF (T,2.5,3)
   38 CONTINUE
      X = PLOTF (0.0,2.5,4)
      X = PLOTF (0.0,0.0,3)
      DO 14 I = 1,111
   14 A(I)=N(I)
      A(1) = 0.3
      X = PLOTF (SN,SU,1)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF(A(1),U(1),3)
   13 DO 15 I=2,110
      X=PLOTF(A(I),U(I),4)
   15 CONTINUE
      X = PLOTF(0.0,0.0,3)
      X = PLOTF(SN,SRHO,1)
      X = PLOTF(0.0,0.0,2)
      K = 0
   18 I1 = 2 + 11*K
      I2 = 9 + I1
   16 DO 17 I = I1,I2
      X = PLOTF(A(I),RHO(I),3)
   17 CONTINUE
      K = K + 1
      IF (K - 10) 18,19,19
   19 REF = 0.0
      X = PLOTF(0.0,0.0,3)
      X = PLOTF(SN,SZ,1)
   20 X = PLOTF (0.0,REF,2)
      X = PLOTF(A(1),Z(1),3)
      K = 0
   21 I1 = 2 + 11*K
      I2 = 9 + I1
      DO 22 I = I1,I2
      X = PLOTF (A(I),Z(I),3)
   22 CONTINUE
      K = K + 1
      IF (K - 10) 21,23,23
   23 X = PLOTF (0.0,REF,3)
      X=PLOTF(SN,SFLUX,1)
```

```
      X = PLOTF(0.0,REF,2)
      X = PLOTF (A(1), FLUX(1),3)
      K = 0
   24 I1 = 2 + 11*K
      I2 = 9 + I1
      DO 25 I = I1,I2
      X = PLOTF(A(I),FLUX(I),3)
   25 CONTINUE
      K = K + 1
      IF(K - 10) 24,26,26
   26 X=PLOTF(0.0,REF,3)
      X = PLOTF(1.0,1.0,1)
      X = PLOTF(0.0,0.0,2)
      X = PLOTF (-5.0,17.5,3)
   27 CONTINUE
      GO TO 9
      END
```

6. ANALYTIC ESTIMATOR SOLUTIONS

```
 1 FORMAT (6F16.8)
 2 FORMAT (1H1)
   D = 0.31
   PRINT 2
   DO 10 N = 1,200
   RHO = -1.01 + 0.01 * N
   Z = EXPF(D * RHO/(1.0 - RHO))
   Y = Z/(1.0 - RHO)
   RHO1 = LOGF(Y)/(D+LOGF(Y))
   Z1 = 1.0 - RHO1
   Z2 = Z1 * EXPF(D*RHO1/(1.0 - RHO1))
10 PRINT 1, RHO, Z, Y, RHO1, Z1, Z2
   END
```

7. FINITE DIFFERENCE SYSTEM WITH ESTIMATOR AND CONTROL

```
      DIMENSION ALPHA(2),          X1(4), X2(4),  HTH1(2,2),
     1F(2,2), H2(2), HTH2(2,2), C2INV(2,2),
     2X1ERR(2),X2ERR(2), XBAR(4),H1(2), C2(2,2),
     3H2F(2),HFTHF(2,2),H2FTYE(2),HTH1DX(2),VECTOR(2),DELTX1(2)
     4,X1TEMP(2),X1DIFF(2),FX1DIF(2), Y2(4)
   10 FORMAT(1H1,28X,18HTRANSIENT RESPONSE,30X,4HB1 =,F14.8,5X,
     14HB2 =, F14.8)
   11 FORMAT (1H )
   12 FORMAT (5X,4(3X,F16.8),27X,I3)
   13 FORMAT (F14.8)
   14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX/)
   15 FORMAT (1H ,I4,4(3X,F16.8))
   16 FORMAT (I5,4F14.8)
   17 FORMAT (I5,F14.8)
   18 FORMAT (I5)
   19 FORMAT (4F16.8)
      READ 18, M
      READ 13, T
      READ 13, ALAMBD
      READ 13, Z0
      READ 19, B1, B2, A,C
   20 READ 13, RHO0
      PRINT 10, B1, B2
      PRINT 11
      PRINT 14
      EPS1 = 1.0E-6
      N = 0
      PI = 0.
      FLUX = Z0/(1.0 - RHO0)
      U = 0.0
      UT = 0.0
      UL = 1.0
      YK = FLUX
      D = ALAMBD
      ALPHA(1) = 1.0
      ALPHA(2) = 0.0
      PRINT 15, N, RHO0, Z0, FLUX
      DELTAZ = Z0 - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO0, DELTAZ, DELTAN
      RHOK = RHO0
      ZK = Z0
      DELT = 0.1
      H1(1) = 1.0
      H1(2) = 1.0
      HTH1(1,1) = 1.0
      HTH1(1,2) = 1.0
      HTH1(2,1) = 1.0
```

```
      HTH1(2,2) = 1.0
   21 DO 22 K = 1,10
      IF (U) 25,24,25
   24 Z = ZK * EXPF((ALAMBD * RHOK * K * 0.1 * T)/(1.0 - RHOK))
      GO TO 26
   25 Z=ZK*EXPF((ALAMBD/U)*LOGF((1.0-RHOK)/(1.0-RHOK-U*K*0.1*T)
     1)-ALAMBD*K*0.1*T)
   26 RHO = RHOK + U*K*0.1*T
      FLUX = Z/(1.0-RHO)
      N = N+1
      PRINT 15, N, RHO, Z, FLUX
      DELTAZ = Z - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   22 CONTINUE
      ZK = Z
      RHOK = RHO
      YK = FLUX
      L = 1
      X1(1) = ALPHA(1)
      X1(2) = ALPHA(2)
      IF (U) 28,23,28
   23 EXPO = EXPF(D*T*X1(2)/(1.0 - X1(2)))
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2)
      GO TO 27
   28 EXPO = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2) + U * T
      GO TO 29
   27 EXPO = EXPF(D*T*X1(2)/(1.0 - X1(2)))
      XBAR(1) = X1(1) * EXPO
      XBAR(2) = X1(2)
      F(1,1) = EXPO
      F(1,2) = (D*T*X1(1)/(1.0 - X1(2))**2)*EXPO
      GO TO 30
   29 EXPO = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      XBAR(1) = X1(1) * EXPO
      XBAR(2) = X1(2) + U * T
      F(1,1) = EXPO
      F(1,2) = D*T*X1(1)*EXPO/((1.0-X1(2))*(1.0-X1(2)-U*T))
   30 F(2,1) = 0.0
      F(2,2) = 1.0
      H2(1) = 1.0/(1.0 - X2(2))
      H2(2) = X2(1)/(1.0 - X2(2))**2
      DO 31 I = 1,2
      DO 31 J = 1,2
   31 HTH2(I,J) = H2(I) * H2(J)
      DO 32 I = 1,2
      H2F(I) = 0.0
      DO 32 J = 1,2
   32 H2F(I) = H2F(I) + H2(J) * F(J,I)
      DO 33 I=1,2
      DO 33 J = 1,2
```

```
33 HFTHF(I,J) = H2F(I) * H2F(J)
   DO 34 I = 1,2
   DO 34 J = 1,2
34 C2INV(I,J) = HTH1(I,J) + HFTHF(I,J)
   DEN2 = C2INV(1,1)*C2INV(2,2) - C2INV(1,2)*C2INV(2,1)
   C2(1,1) = C2INV(2,2)/DEN2
   C2(1,2) = -C2INV(1,2)/DEN2
   C2(2,1) = -C2INV(2,1)/DEN2
   C2(2,2) = C2INV(1,1)/DEN2
   DO 35 I = 1,2
35 X1ERR(I) = ALPHA(I) - X1(I)
   HX1ERR = 0.0
   DO 36 I = 1,2
36 HX1ERR = HX1ERR + H1(I) * X1ERR(I)
   PI1 = HX1ERR**2
   DO 37 I = 1,2
37 X2ERR(I) = X2(I) - XBAR(I)
   YH = X2(1)/ (1.0 - X2(2))
   YERR1 = YK - YH
   PI2 = YERR1 **2
   HX2ERR = 0.0
   DO 38 I = 1,2
38 HX2ERR = HX2ERR + H2(I) * X2ERR(I)
   YERR = YERR1 + HX2ERR
   DO 39, I = 1,2
39 H2FTYE(I) = H2F(I) * YERR
   DO 40 I = 1,2
40 HTH1DX(I) = H1(I) * HX1ERR
   DO 41 I = 1,2
41 VECTOR (I) = H2FTYE(I) + HTH1DX(I)
   DO 42 I = 1,2
   DELTX1(I) = 0.0
   DO 42 J = 1,2
42 DELTX1(I) = DELTX1(I) + C2(I,J) * VECTOR(J)
   DO 43, I = 1,2
43 X1TEMP(I) = X1(I)
   DO 44, I = 1,2
44 X1(I) = X1(I) + DELTX1(I)
   IF(X1(2) - 0.8) 46,46,45
45 X1(1) = X1TEMP(1) + DELTX1(1) * (0.8 - U*T - X1TEMP(2))/
  1DELTX1(2)
   X1(2) = 0.8 - U * T
46 CONTINUE
   DO 47, I = 1,2
47 X1DIFF(I) = X1(I) - X1TEMP(I)
   DO 48 I = 1,2
   FX1DIF(I) = 0.0
   DO 48 J = 1,2
48 FX1DIF(I) = FX1DIF(I) + F(I,J) * X1DIFF(J)
   DO 49 I = 1,2
49 X2(I) = XBAR(I) + FX1DIF(I)
   PI = PI1 + PI2
   IF (PI - EPS1  ) 51,51,50
50 L = L + 1
```

```
      IF (U) 29,27,29
   51 YH = X2(1)/ (1.0 - X2(2))
      U  = B1 * (X2(1) - 1.0) + B2 * X2(2)
      PRINT 12, U,   X2(2), X2(1), YH, L
      PUNCH 17,N,U
      DO 56 I = 1,2
      DO 56 J = 1,2
   56 HTH1(I,J) = HTH2(I,J)
      DO 57 I = 1,2
   57 ALPHA(I) = X2(I)
      DO 58 I = 1,2
      H1(I) = H2(I)
   58 X1(I) = X2(I)
      IF (U) 61,60,61
   60 EXPO = EXPF(D*T*X1(2) /(1.0 - X1(2)))
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2)
      GO TO 62
   61 EXPO = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2) + U * T
   62 UT = B1 * (X2(1) - 1.0) + B2 * X2(2)
      IF(UT - UL) 53,52,52
   52 UT = UL
      GO TO 55
   53 IF(UT + UL) 54,55,55
   54 UT = -UL
   55 IF (N-M) 21,59,59
   59 CONTINUE
      GO TO 20
      END
```

8. DIFFERENTIAL SYSTEM WITH ESTIMATOR AND CONTROL

```
      COMMON U, H, EPS2
      DIMENSION ALPHA(2),          X1(4), X2(4),  HTH1(2,2),
     1F(2,2), H2(2), HTH2(2,2), C2INV(2,2),
     2X1ERR(2),X2ERR(2), XBAR(4),H1(2), C2(2,2),
     3H2F(2),HFTHF(2,2),H2FTYE(2),HTH1DX(2),VECTOR(2),DELTX1(2)
     4,X1TEMP(2),X1DIFF(2),FX1DIF(2), Y2(4)
   10 FORMAT(1H1,28X,18HTRANSIENT RESPONSE,30X,4HB1 =,F14.8,5X,
     14HB2 =, F14.8)
   11 FORMAT (1H )
   12 FORMAT (5X,4(3X,F16.8),27X,I3)
   13 FORMAT (F14.8)
   14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX/)
   15 FORMAT (1H ,I4,4(3X,F16.8))
   16 FORMAT (I5,4F14.8)
   17 FORMAT (I5,F14.8)
   18 FORMAT (I5)
   19 FORMAT (4F16.8)
      READ 18, M
      READ 13, T
      READ 13, ALAMBD
      READ 13, Z0
      READ 19, B1, B2, A,C
   20 READ 13, RHO0
      PRINT 10, B1, B2
      PRINT 11
      PRINT 14
      EPS1 = 1.0E-6
      EPS2 = 1.0E-4
      N = 0
      P1 = 0.
      FLUX = Z0/(1.0 - RHO0)
      U = 0.0
      UT = 0.0
      UL = 1.0
      YK = FLUX
      D = ALAMBD
      ALPHA(1) = 1.0
      ALPHA(2) = 0.0
      PRINT 15, N, U, RHO0, Z0, FLUX
      DELTAZ = Z0 - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO0, DELTAZ, DELTAN
      RHOK = RHO0
      ZK = Z0
      DELT = 0.1
      H1(1) = 1.0
      H1(2) = 1.0
      HTH1(1,1) = 1.0
```

```
      HTH1(1,2) = 1.0
      HTH1(2,1) = 1.0
      HTH1(2,2) = 1.0
   21 DO 22 K = 1,10
      IF (U) 25,24,25
   24 Z = ZK * EXPF((ALAMBD * RHOK * K * 0.1 * T)/(1.0 - RHOK))
      GO TO 26
   25 Z=ZK*EXPF((ALAMBD/U)*LOGF((1.0-RHOK)/(1.0-RHOK-U*K*0.1*T)
     1)-ALAMBD*K*0.1*T)
   26 RHO = RHOK + U*K*0.1*T
      FLUX = Z/(1.0-RHO)
      N = N+1
      PRINT 15,N,U,RHO,Z,FLUX
      DELTAZ = Z - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   22 CONTINUE
      WRITE TYPE 11
      ZK = Z
      RHOK = RHO
      YK = FLUX
      L = 1
      X1(1) = ALPHA(1)
      X1(2) = ALPHA(2)
      X1(3) = 1.0
      X1(4) = 0.0
      CALL INTEGR (X1, X2)
   29 X1(3) = 1.0
      X1(4) = 0.0
      CALL INTEGR (X1, XBAR)
      F(1,1) = XBAR(3)
      F(1,2) = XBAR(4)
   30 F(2,1) = 0.0
      F(2,2) = 1.0
      H2(1) = 1.0/(1.0 - X2(2))
      H2(2) = X2(1)/(1.0 - X2(2))**2
      DO 31 I = 1,2
      DO 31 J = 1,2
   31 HTH2(I,J) = H2(I) * H2(J)
      DO 32 I = 1,2
      H2F(I) = 0.0
      DO 32 J = 1,2
   32 H2F(I) = H2F(I) + H2(J) * F(J,I)
      DO 33 I=1,2
      DO 33 J = 1,2
   33 HFTHF(I,J) = H2F(I) * H2F(J)
      DO 34 I = 1,2
      DO 34 J = 1,2
   34 C2INV(I,J) = HTH1(I,J) + HFTHF(I,J)
      DEN2 = C2INV(1,1)*C2INV(2,2) - C2INV(1,2)*C2INV(2,1)
      C2(1,1) = C2INV(2,2)/DEN2
      C2(1,2) = -C2INV(1,2)/DEN2
      C2(2,1) = -C2INV(2,1)/DEN2
      C2(2,2) = C2INV(1,1)/DEN2
```

```
      DO 35 I = 1,2
   35 X1ERR(I) = ALPHA(I) - X1(I)
      HX1ERR = 0.0
      DO 36 I = 1,2
   36 HX1ERR = HX1ERR + H1(I) * X1ERR(I)
      PI1 = HX1ERR**2
      DO 37 I = 1,2
   37 X2ERR(I) = X2(I) - XBAR(I)
      YH = X2(1)/ (1.0 - X2(2))
      YERR1 = YK - YH
      PI2 = YERR1 **2
      HX2ERR = 0.0
      DO 38 I = 1,2
   38 HX2ERR = HX2ERR + H2(I) * X2ERR(I)
      YERR = YERR1 + HX2ERR
      DO 39, I = 1,2
   39 H2FTYE(I) = H2F(I) * YERR
      DO 40 I = 1,2
   40 HTH1DX(I) = H1(I) * HX1ERR
      DO 41 I = 1,2
   41 VECTOR (I) = H2FTYE(I) + HTH1DX(I)
      DO 42 I = 1,2
      DELTX1(I) = 0.0
      DO 42 J = 1,2
   42 DELTX1(I) = DELTX1(I) + C2(I,J) * VECTOR(J)
      DO 43, I = 1,2
   43 X1TEMP(I) = X1(I)
      DO 44, I = 1,2
   44 X1(I) = X1(I) + DELTX1(I)
      IF(X1(2) - 0.8) 46,46,45
   45 X1(1) = X1TEMP(1) + DELTX1(1) * (0.8 - U*T - X1TEMP(2))/
     1DELTX1(2)
      X1(2) = 0.8 - U * T
   46 CONTINUE
      DO 47, I = 1,2
   47 X1DIFF(I) = X1(I) - X1TEMP(I)
      DO 48 I = 1,2
      FX1DIF(I) = 0.0
      DO 48 J = 1,2
   48 FX1DIF(I) = FX1DIF(I) + F(I,J) * X1DIFF(J)
      DO 49 I = 1,2
   49 X2(I) = XBAR(I) + FX1DIF(I)
      PI = PI1 + PI2
      IF (PI - EPS1  ) 51,51,50
   50 L = L + 1
      WRITE TYPE 18, L
      GO TO 29
   51 YH = X2(1)/ (1.0 - X2(2))
      U = UT
      PRINT 12, U, X2(2), X2(1), YH, L
      PUNCH 17,N,U
      DO 56 I = 1,2
      DO 56 J = 1,2
   56 HTH1(I,J) = HTH2(I,J)
```

```
      DO 57 I = 1,2
   57 ALPHA(I) = X2(I)
      DO 58 I = 1,2
      H1(I) = H2(I)
   58 X1(I) = X2(I)
      CALL INTEGR(X1, X2)
   62 UT = B1 * (X2(1) - 1.0) + B2 * X2(2)
      IF(UT - UL) 53,52,52
   52 UT = UL
      GO TO 55
   53 IF(UT + UL) 54,55,55
   54 UT = -UL
   55 IF (N-M) 21,59,59
   59 GO TO 20
      END

      SUBROUTINE INTEGR (X1, Y2)
      COMMON U, H, EPS2
      DIMENSION Y0(4), Y1(4), Y2(4), F0(4), F1(4), F2(4),
     1ERROR(4), F(4), Y(4), X1(4)
    1 FORMAT (F14.8)
      N = 4
      H = 1.0
      DO 30 I=1,N
   30 Y0(I) = X1(I)
   39  LOC = 0
      MLOC = 1
   38 HA = .33333333*H
      HB = .16666667*H
      HC = .125*H
      HD = .375*H
      HE = .5*H
      HF = 1.5*H
      HG = 2.*H
      HH = .66666667*H
      WRITE TYPE 1, H
   48  CALL FCT (Y0, F0)
      DO 41 I=1,N
   41  Y1(I) = Y0(I) + HA * F0(I)
      CALL FCT (Y1, F1)
      DO 42 I=1,N
   42  Y1(I) = Y0(I) + HB*F0(I) + HB*F1(I)
      CALL FCT (Y1, F2)
      DO 44 I=1,N
   44  Y1(I) = Y0(I) + HE*F0(I) - HF*F1(I) + HG*F2(I)
      CALL FCT (Y1, F1)
      DO 45 I=1,N
   45 Y2(I) = Y0(I) + HB*F0(I) + HH*F2(I) + HB*F1(I)
      DO 34 I=1,N
      ERROR(I) = .2 * ABSF(Y1(I) - Y2(I))
      IF (EPS2- ERROR(I)) 35, 34, 34
   34 CONTINUE
      DO 32 I=1,N
   32 Y0(I) = Y2(I)
```

```
      LOC = LOC + 1
   33 IF (LOC - MLOC) 37, 99, 99
8  37 IF (LOC * 1) 48, 47, 48
   47 IF (MLOC - 2) 48, 49, 49
   49 DO 31 I=1,N
      IF (EPS2- ERROR(I) * 64.) 48, 48, 31
   31 CONTINUE
   24 H = HG
      LOC = LOC / 2
      MLOC = MLOC / 2
      GO TO 38
   35 H = HE
      MLOC = MLOC * 2
      LOC = LOC * 2
      GO TO 38
   99 RETURN
      END

      SUBROUTINE FCT (Y, F)
      COMMON U
      DIMENSION Y(4), F(4)
      F(1) = 0.31  * Y(1) * Y(2) /(1.0 - Y(2))
      F(2) = U
      F(3) = 0.31  * Y(3) * Y(2)/(1.0 - Y(2))
      F(4) = 0.31 *(Y(2)*Y(4)+Y(1)/(1.0-Y(2)))/(1.0-Y(2))
      RETURN
      END
```

REFERENCES CITED

1. *Nuclear Reactors Built, Being Built, or Planned in the United States as of June 30, 1968*, TID-8200 (18th Rev.) (June 1968).

2. *Liquid Metal Fast Breeder Reactor (LMFBR) Program Plan, Vol. 4: Instrumentation and Control*, WASH-1104 (Aug. 1968).

3. N. Kallay, *Dynamic Programming and Nuclear Reactor Systems Design*, Nucl. Sci. Eng., 8, No. 4, 315-325 (Oct. 1960).

4. *On-line Computers for Power Reactors*, Nucleonics, 20, No. 6, 51-74 (June 1962).

5. M. A. Schultz and F. C. Legler, *Application of Digital Computer Techniques to Reactor Operation*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 321-330.

6. N. Wiener, *The Extrapolation, Interpolation and Smoothing of Stationary Time Series*, (New York: John Wiley & Sons, Inc., 1949).

7. R. Bellman, *The Theory of Dynamic Programming*, Bull. Amer. Math. Soc., 60, 503-516 (1954).

8. V. G. Boltyanskii, R. V. Gamkrelidze, and L. S. Pontryagin, *A Note on the Theory of Optimal Processes*, Dokl. Akad. Nauk SSSR, 110, No. 1, 7-10 (1956) (in Russian).

9. A. E. Foureau, *Optimal Non-linear Automatic Start-up Program for a Nuclear Reactor at Low Power*, Revue A Tijdschrift, 4, No. 4, 173-183 (1962) (in French).

10. C. N. Shen and F. G. Haag, *Computer Programs for Optimum Start-up of Nuclear Propulsion Systems*, TID-16730 (July 1962).

11. C. N. Shen and F. G. Haag, *Application of Optimum Control to Nuclear Reactor Start-up*, IEEE Trans. Nucl. Sci., NS11, No. 2, 1-9 (Apr. 1964).

12. C. N. Shen and F. G. Haag, *Adaptive Control for Nuclear Reactor Start-up and Regulation*, Trans. Amer. Nucl. Soc., 6, No. 1, 109-110 (June 1963).

13. F. G. Haag, *An Investigation of Optimum Control for Nuclear Reactors*, Ph.D. Thesis, Rensselear Polytechnic Institute, 1964, (University Microfilms No. 65-6458).

14. C. N. Shen and F. G. Haag, *Optimum Nuclear Rocket Start-up to Develop Full Power at Exact Time with Consideration of Noise*, Proc. I & M Sect. Automat. Contr. (Stockholm, Sweden, Sept. 1964), (New York: Academic Press, 1965), 52-65.

15. T. P. Mulcahey, *Time-Optimum Control of Nuclear Reactors with Velocity-limited Control Devices*, Ph.D. Thesis, Purdue Univ., 1963, (UM No. 63-6523); Published as ANL-6695 (Oct. 1963).

REFERENCES CITED

16. T. P. Mulcahey, *Time-optimum Control of Nuclear Reactors with Velocity-limited Control Devices*, Reactor Kinetics and Control, TID-7662 (Apr. 1964), 278-298.

17. Z. R. Rosztoczy, A. P. Sage, and L. E. Weaver, *Application of Pontryagin's Maximum Principle to Flux State Changes in Nuclear Reactors*, Reactor Kinetics and Control, TID-7662 (Apr. 1964), 265-277.

18. Z. R. Rosztoczy, *Optimization Studies in Nuclear Engineering*, Ph.D. Thesis, Univ. Arizona, 1964, (UM No. 64-10,458).

19. A. L. Ruiz, *An Approach to Reactor Control by Dynamic Optimization*, Proc. IEEE Electro-nuclear Conference (Richland, Wash., Apr. 1963), Paper No. CPA63-5150.

20. M. Ash, *Bang-bang Control of Boiling-moderator Nuclear Reactors*, Nucl. Sci. Eng., 16, No. 2, 208-212 (June 1963).

21. R. J. Hermsen, *Optimal Control Analysis of Nuclear Reactors*, Ph.D. Thesis, Univ. Wisconsin, 1963, (UM No. 64-644).

22. I. Kliger, *An Adaptive Automatic Control of a Nuclear Reactor for a Minimal Time Response*, Trans. Amer. Nucl. Soc., 7, No. 1, 59-60 (June 1964).

23. R. R. Mohler, *Optimal Control of Nuclear Reactor Systems*, Trans. Amer. Nucl. Soc., 7, No. 1, 58-59 (June 1964).

24. R. R. Mohler, *Optimal Control of Nuclear Reactor Processes*, Ph.D. Thesis, Univ. Michigan, 1965, (UM No. 65-11,002); Published as LA-3257-MS (Mar. 1965).

25. R. R. Mohler, *Optimal Nuclear-rocket-engine Control*, Neutron Dynamics and Control, CONF-650413 (May 1966), 137-163.

26. L. E. Weaver et al., *Research in and Application of Modern Automatic Control Theory to Nuclear Rocket Dynamics and Control. Progress Report Feb. 1 - Sept. 1, 1964*, NASA-CR-193 (Mar. 1965).

27. P. A. Secker and L. E. Weaver, *Synthesis of Optimal Feedback Control for Nuclear-rocket-engine Start-up*, Neutron Dynamics and Control, CONF-650413 (May 1966), 164-181.

28. P. A. Secker and L. E. Weaver, *Synthesis of Optimal Reactor Feedback Control Using the First Variation*, Trans. Amer. Nucl. Soc., 8, No. 1, 234-235 (June 1965).

29. J. L. Melsa, *Closed-loop, Sub-optimal Control Employing the Second Method of Liapunov*, Ph.D. Thesis, Univ. Arizona, 1965, (UM No. 65-5390); Published as NASA-CR-62187 (Mar. 1965).

REFERENCES CITED

30. J. L. Melsa, *A Closed-loop, Approximately Time-optimal Control Method,* Neutron Dynamics and Control, CONF-650413 (May 1966), 207-224.

31. I. Kliger, *Synthesis of an Optimal Nuclear Reactor Control System,* Neutron Dynamics and Control, CONF-650413 (May 1966), 110-136.

32. E. Duncombe, *On-Line Optimization of Nuclear Reactor Load Control in the Presence of Nonlinearities,* Ph.D. Thesis, Univ. Pittsburgh, 1965, (UM No. 66-8819).

33. E. Duncombe, *Control Systems for Optimal Response to Changes in Load Demand of a Nuclear Reactor Plant,* Trans. Amer. Nucl. Soc., 10, No. 1, 367-368 (June 1967).

34. E. Duncombe and D. E. Rathbone, *Optimization of the Response of a Nuclear Reactor Plant to Changes in Demand,* Proc. JACC, 834-845 (1968).

35. K. Monta and C. G. Lennox, *A Time-optimal, On-off, Program for the NRU Reactor Digital Computer Control Experiment,* J. Nucl. Sci. Technol., 2, No. 10, 391-405 (Oct. 1965).

36. C. A. Desoer and J. Wing, *An Optimal Strategy for a Saturating Sampled-data System,* IRE Trans. Automat. Contr., 6, 5-15 (1961).

37. I. Kliger, *Synthesis of an Optimal Control System for Nuclear Reactors with Generalized Temperature Feedback,* Nuclear Electronics (Vienna: International Atomic Energy Agency, 1966), 443-460.

38. N. M. Sokolova, *Solution of the Problem of the Analytical Design of an Optimum Control for a Nuclear Power Plant,* ANL-Trans-470 (Apr. 1967).

39. L. E. Weaver et al., *Research in and Application of Modern Automatic Control Theory to Nuclear Rocket Dynamics and Control. Semiannual Status Report. Vol. I,* NASA-CR-70862 (Feb. 1966).

40. W. T. Higgins, *The Stability of Certain Nonlinear Time-Varying Systems of Automatic Control,* Ph.D. Thesis, Univ. Arizona, 1966, (UM No. 66-6893); Published as NASA-CR-74134 (Mar. 1966).

41. W. T. Higgins and D. G. Schultz, *Research in and the Application of Modern Automatic Control Theory to Nuclear Rocket Dynamics and Control. Semiannual Status Report. Vol. II,* NASA-CR-74134 (Mar. 1966).

42. I. Kliger, *An Automatic Control of a Nuclear Reactor for a Minimal Time Response,* Proc. IFAC, 1; Bk2, Paper No. 26C (1966).

REFERENCES CITED

43. K. Monta, *Time Optimal Digital Computer Control of Nuclear Reactors: I, Continuous Time System*, J. Nucl. Sci. Technol., 3, No. 6, 227-236 (June 1966).

44. K. Monta, *Time Optimal Digital Computer Control of Nuclear Reactors: II, Discrete Time System*, J. Nucl. Sci. Technol., 3, No. 10, 418-429 (Oct. 1966).

45. K. Monta, *Time Optimal Digital Computer Control of Nuclear Reactors: III, Experiment*, J. Nucl. Sci. Technol., 4, No. 2, 51-62 (Feb. 1967).

46. J. T. Humphries, *Model Reference Adaptive Control of a Nuclear Rocket Engine*, Ph.D. Thesis, Univ. Florida, 1966, (UM No. 66-11,119).

47. J. T. Humphries, R. E. Uhrig, and A. P. Sage, *Model Reference Adaptive Control of a Nuclear Rocket Engine*, Proc. JACC, 492-499 (1966).

48. J. K. Saluja, *Computational Techniques and Performance Criteria for the Optimum Control of Nuclear System Dynamics*, Ph.D. Thesis, Univ. Florida, 1966, (UM No. 67-13,160).

49. J. Saluja, A. P. Sage, and R. E. Uhrig, *Optimum Open and Closed-loop Control of Nuclear System Dynamics*, Trans. Amer. Nucl. Soc., 9, No. 2, 462-463 (Oct. 1966).

50. T. W. Ellis, *Sequential Suboptimal Adaptive Control of Nonlinear Systems*, Ph.D. Thesis, Univ. Florida, 1966, (UM No. 67-12,922).

51. A. P. Sage and T. W. Ellis, *Sequential Suboptimal Adaptive Control of Nonlinear Systems*, Proc. NEC, 22, 692-697 (1966).

52. G. W. Masters, *Sequential State and Parameter Estimation in Discrete Nonlinear Systems*, Ph.D. Thesis, Univ. Florida, 1966, (UM No. 67-3483).

53. A. P. Sage and G. W. Masters, *Identification and Modeling of States and Parameters of Nuclear Reactor Systems*, IEEE Trans. Nucl. Sci., NS-14, No. 1, 279-285 (Feb. 1967).

54. Y. Ogawa, I. Kaji, and Y. Ozawa, *Time Optimum Control for Reactors with Two Kinds of Internal Feedback*, J. Nucl. Sci. Technol., 3, No. 11, 455-464 (Nov. 1966).

55. M. Rasetti and M. Vallauri, *Application to the Nuclear Field of New Methods of the Theory of Automatic Control*, Energ. Nucl. (Milan), 13, No. 12, 668-679 (Dec. 1966).

REFERENCES CITED

56. N. Tataru, T. Bajenescu, and S. Ghetaru, *A Self-optimizing System for the Power Control of Nuclear Reactors*, Rev. Roumaine Sci. Tech. Electrotech. Energet., 11, No. 1, 33-42 (1966).

57. C. L. Partain, *A Direct Digital Control Study for Nuclear Reactor Systems*, Ph.D. Thesis, Purdue Univ., 1967, (UM No. 67-10,237).

58. C. L. Partain and R. E. Bailey, *A Direct Digital-control Algorithm for Nuclear Reactor Systems*, Trans. Amer. Nucl. Soc., 10, No. 1, 368 (June 1967).

59. C. L. Partain and R. E. Bailey, *An Application of Time Optimal Control Theory to the Nuclear Reactor Startup Problem*, Proc. JACC, 71-80 (1967).

60. J. W. Herring, *Design of Nonlinear Control Systems via State Variable Feedback*, Ph.D. Thesis, Univ. Arizona, 1967, (UM No. 67-11,361).

61. J. W. Herring et al., *Design of Linear and Nonlinear Control Systems via State Variable Feedback, with Applications in Nuclear Reactor Control*, NASA-CR-82516 (Feb. 1967).

62. L. W. Weaver, *State Variable Feedback Design of Reactor Control Systems*, Trans. Amer. Nucl. Soc., 10, No. 1, 205 (June 1967).

63. K. Miyazaki, *Optimization of Reactor Control Systems by Wiener's Theory*, J. Nucl. Sci. Technol., 4, No. 3, 107-114 (Mar. 1967).

64. L. J. Habegger, *The Use of the Kalman Filter for the Estimation of Nuclear Reactor Parameters*, Ph.D. Thesis, Purdue Univ., 1968.

65. L. J. Habegger, R. E. Bailey, and K. Kadavanich, *The Use of Quasi-linearization for the Identification of Nuclear Reactor Parameters*, Trans. Amer. Nucl. Soc., 10, No. 1, 176-177 (June 1967).

66. J. L. Melsa et al., *Research in and Application of Modern Automatic Control Theory to Nuclear Rocket Dynamics and Control*, NASA-CR-87475 (July 1967).

67. R. R. Mohler, *Modern Control Applications to Nuclear Propulsion*, SC-DC-67-1916 (July 1967).

68. L. E. Weaver and R. E. Vanasse, *State Variable Feedback Control of Multiregion Reactors*, Nucl. Sci. Eng., 29, No. 2, 264-271 (Aug. 1967).

69. R. R. Mohler and H. J. Price, *Optimal Nuclear Reactor Control*, SC-CR-67-2746 (Aug. 1967).

REFERENCES CITED

70. H. J. Price and R. R. Mohler, *Computation of Optimal Controls for a Nuclear Rocket*, SC-CR-67-2723 (Sept. 1967).

71. T. J. Marciniak, *Time-optimal Digital Control of Zero Power Nuclear Reactors*, Ph.D. Thesis, Univ. Notre Dame, 1968.

72. S. Glasstone and M. C. Edlund, *The Elements of Nuclear Reactor Theory*, (Princeton, N.J.: D. Van Nostrand Co., Inc., 1952).

73. A. M. Weinberg and E. P. Wigner, *The Physical Theory of Neutron Chain Reactors*, (Chicago: The University of Chicago Press, 1958).

74. R. V. Meghreblian and D. K. Holmes, *Reactor Analysis*, (New York: McGraw-Hill Book Co., 1960).

75. H. S. Isbin, *Introductory Nuclear Reactor Theory*, (New York: Reinhold Publishing Corp., 1963).

76. M. Ash, *Nuclear Reactor Kinetics*, (New York: McGraw-Hill Book Co., 1965).

77. A. Radkowsky (ed.), *Naval Reactor Physics Handbook, I: Selected Basic Techniques*, TID-7030 (1964).

78. E. P. Gyftopolous, *General Reactor Dynamics*, Chapter 3, The Technology of Nuclear Reactor Safety, 1, Reactor Physics and Control, (eds.) T. J. Thompson and J. G. Bekerley (Cambridge, Mass., M.I.T. Press, 1964), 175-204.

79. W. J. McCarthy and D. Okrent, *Fast Reactor Kinetics*, Chapter 10, The Technology of Nuclear Reactor Safety, 1, Reactor Physics and Control, (eds.) T. J. Thompson and J. G. Bekerley (Cambridge, Mass., M.I.T. Press, 1964), 530-607.

80. J. M. Harrer, *Nuclear Reactor Control Engineering*, (Princeton, N.J.: D. Van Nostrand Co., Inc., 1963).

81. G. R. Keepin, *Physics of Nuclear Kinetics*, (Reading, Mass.: Addison-Wesley Publishing Co., 1965).

82. M. A. Schultz, *Control of Nuclear Reactors and Power Plants*, (2nd ed; New York: McGraw-Hill Book Co., 1961).

83. L. E. Weaver, *System Analysis of Nuclear Reactor Dynamics*, (New York: Rowman and Littlefield, Inc., 1963).

84. L. E. Weaver, *Reactor Dynamics and Control: State Space Techniques*, (New York: American Elsevier Publishing Co., Inc., 1968).

85. *Reactor Physics Constants*, ANL-5800, 2nd ed. (July 1963).

REFERENCES CITED

86. P. Henrici, *Elements of Numerical Analysis* (New York: John Wiley & Sons, Inc., 1964).

87. L. A. Zadeh and C. A. Desoer, *Linear System Theory; The State Space Approach*, (New York: McGraw-Hill Book Co., 1963).

88. P. M. DeRusso, R. J. Roy, and C. M. Close, *State Variables for Engineers*, (New York: John Wiley & Sons, Inc., 1965).

89. S. C. Gupta, *Transform and State Variable Methods in Linear Systems*, (New York: John Wiley & Sons, Inc., 1966).

90. K. Ogata, *State Space Analysis of Control Systems*, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1966).

91. L. K. Timothy and B. E. Bona, *State Space Analysis*, (New York: McGraw-Hill Book Co., 1968).

92. C. Chen and I. Haas, *Elements of Control Systems Analysis; Classical and Modern Approaches*, (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1968).

93. R. E. Bellman, *Dynamic Programming*, (Princeton, N.J.: Princeton University Press, 1957).

94. R. Bellman and R. Kalaba, *Dynamic Programming and Modern Control Theory*, (New York: Academic Press, 1965).

95. S. E. Dreyfus, *Dynamic Programming and the Calculus of Variations*, (New York: Academic Press, 1965).

96. J. T. Tou, *Optimum Design of Digital Control Systems*, (New York: Academic Press, 1963).

97. J. T. Tou, *Modern Control Theory*, (New York: McGraw-Hill Book Co., (1964).

98. L. Lapidus and R. Luus, *Optimal Control of Engineering Processes*, (Waltham, Mass.: Blaisdell Publishing Co., 1967).

99. H. Freeman, *Discrete-time Systems: An Introduction to the Theory*, (New York: John Wiley & Sons, Inc., 1965).

100. D. P. Lindorff, *Theory of Sampled-Data Control Systems*, (New York: John Wiley & Sons, Inc., 1965).

101. T. J. Marciniak, *Time-optimal Digital Control of Zero-power Nuclear Reactors*, Trans. Amer. Nucl. Soc., 11, No. 2, 633 (Nov. 1968).

REFERENCES CITED

102. J. H. Price and R. R. Johler, *How to Use Quasi-linear Programming for Calculating Optimum Reactor Control,* Reactor Fuel-Process. Technol., 11, No. 3, 123-126 (Summer 1968).

103. A. M. Legendre, *New Methods for the Determination of the Orbits of Comets,* (Paris, 1806) (in French).

104. R. E. Kalman, *A New Approach to Linear Filtering and Prediction Problems,* J. Basic Eng., 82, No. 1, 35-45 (Mar. 1960).

105. R. E. Kalman, *On the General Theory of Control Systems,* Proc. IFAC, 1, 481-492 (1960).

106. R. E. Kalman and R. S. Bucy, *New Results in Linear Filtering and Prediction Theory,* J. Basic Engr., 83, No. 1, 95-108 (Mar. 1961).

107. R. E. Kalman, *New Methods and Results in Linear Prediction and Filtering Theory,* Proc. First Symp. Eng. Appl. Random Function Theory and Probability (Purdue Univ., Nov. 1960), (eds.) J. L. Bogdanoff and F. Kozin (New York: John Wiley & Sons, Inc., 1963), 270-388.

108. Y. C. Ho, *The Method of Least Squares and Optimal Filtering Theory,* RM-3329-PR (Oct. 1962).

109. Y. V. Linnik, *Method of Least Squares and Principles of the Theory of Observations,* (New York: Pergamon Press, 1961).

110. R. C. K. Lee, *Optimal Estimation, Identification, and Control,* (Cambridge, Mass.: M.I.T. Press, 1964).

111. R. F. Ohap and A. R. Stubberud, *A Technique for Estimating the State of a Nonlinear System,* IEEE Trans. Automat. Contr., AC-10, No. 2, 150-155 (Apr. 1965).

112. H. Cox, *Recursive Nonlinear Filtering,* Proc. NEC, 21, 770-775, (1965).

113. V. O. Mowery, *Least Squares Recursive Differential-Correction Estimation in Nonlinear Problems,* IEEE Trans. Automat. Contr., AC-10, No. 4, 399-407 (Oct. 1965).

114. R. Deutsch, *Estimation Theory,* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965).

115. R. Sridhar and J. B. Pearson, *Digital Estimation of Nonlinear Processes,* Proc. IEEE Region 6 Annu. Conf. (Tucson, Ariz., 1966).

116. J. Peschon et al., *Research on the Design of Adaptive Control Systems, Vol. 1,* NASA-CR-81724 (July 1966).

REFERENCES CITED

117. J. Peschon et al., *Research on the Design of Adaptive Control Systems, Vol. 2,* NASA-CR-81731 (Sept. 1966).

118. W. B. Phillips, *Estimation and Identification of Nonlinear Sampled Data Systems,* Ph.D. Thesis, Purdue Univ., 1966, (UM No. 67-5482).

119. H. W. Sorenson, *A Nonlinear Perturbation Theory for Estimation and Control of Time-Discrete Stochastic Systems,* NASA-CR-92981 (Jan. 1968), Ph.D. Thesis, Univ. California, 1966, (UM No. 67-453).

120. H. W. Sorenson, *Kalman Filtering Techniques,* Advances in Control Systems, 3, (ed.) C. T. Leondes (New York: Academic Press, Inc., 1966), 219-292.

121. A. P. Sage and G. W. Masters, *Least-squares Curve Fitting and Discrete Optimum Filtering,* IEEE Trans. Educ., E-10, No. 1, 29-36 (Mar. 1967).

122. J. D. Irwin, *Optimum Discrete-Data Estimation,* Ph.D. Thesis, Univ. Tennessee, 1967, (UM No. 67-10,737).

123. J. B. Pearson, *On Nonlinear Least-squares Filtering,* Automatica, 4, No. 3, 97-106 (Aug. 1967).

124. P. B. Liebelt, *An Introduction to Optimal Estimation,* (Reading, Mass.: Addison-Wesley Publishing Co., 1967).

125. A. P. Sage, *Optimum Systems Control,* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1968).

126. R. H. Merson, *An Operational Method for the Study of Integration Processes,* Proc. Symp. Data Processing, Weapons Research Establishment, Salisbury, South Australia (1957), 24-26.

127. L. Fox, *Numerical Solution of Ordinary and Partial Differential Equations,* (Reading, Mass.: Addison-Wesley Publishing Co., 1962).

128. L. Meier, *Combined Optimum Control and Estimation Theory,* NASA-CR-426 (Apr. 1966).

129. G. W. Deley and G. F. Franklin, *Optimal Bounded Control of Linear Sampled-data Systems with Quadratic Loss,* JACC, 404-410 (1964).

130. F. H. Kishi, *A Suboptimal On-line Discrete Controller with Bounded Control Variables,* IEEE Trans. Appl. Ind., 83, No. 73, 216-222 (July 1964).

131. Y. Sawargi and K. Inoue, *A Fundamental Study on the Computing Time Delay in Computer Control Systems,* Kyoto Univ., Eng. Res. Inst. Rep. No. 131 (Jan. 1967).

REFERENCES CITED

132. R. Bellman, *Adaptive Control Processes: A Guided Tour*, (Princeton, N.J.: Princeton University Press, 1960).

SELECTED NUCLEAR BIBLIOGRAPHY

A. Z. Akcasu and A. Dalfes, *A Study of Nonlinear Reactor Dynamics*, Nucl. Sci. Eng., 8, No. 2, 89-93 (Aug. 1960).

A. Z. Akcasu, *Local Stability in Reactors with a Linear Feedback*, Nucl. Sci. Eng., 24, No. 1, 88-89 (Jan. 1966).

A. Z. Akcasu, *A Nonlinear Study of Reactor Systems with Linear Feedback*, Neutron Dynamics and Control, CONF-650413 (May 1966), 28-37.

A. Z. Akcasu and L. D. Noble, *A Nonlinear Study of Reactors with Linear Feedback*, Nucl. Sci. Eng., 25, No. 1, 47-57 (May 1966).

A. Z. Akcasu and L. D. Noble, *Lagrange Stability in Reactors with a Linear Feedback*, Nucl. Sci. Eng., 25, No. 4, 427-429 (Aug. 1966).

R. W. Albrecht, *The Measurement of Dynamic Nuclear Reactor Parameters by Methods of Stochastic Processes*, Trans. Amer. Nucl. Soc., 4, No. 2, 311-312 (Nov. 1961).

R. W. Albrecht, *The Measurement of Dynamic Nuclear Reactor Parameters Using the Variance of the Number of Neutrons Detected*, Nucl. Sci. Eng., 14, No. 2, 153-158 (Oct. 1962).

C. A. Anderson, *On High Frequency Transfer Function Measurements for Coupled Fast Power Reactors*, LA-DC-8299 (Feb. 1967).

C. A. Anderson, *RAPID (Reactor and Plant Integrated Dynamics Computer Program)*, LA-3694 (June 1967).

A. J. Arker and D. G. Lewis, *Rapid Flow Transients in Closed Loops*, TID 7529 (Pt. 1), Bk. 1, 33-47 (Nov. 1967).

M. Ash, *Solutions of the Reactor Kinetics Equations for Time Varying Reactivities*, J. Appl. Phys., 27, No. 9, 1030-1031 (Sept. 1956).

M. Ash, *On Control of Reactor Shut-down Involving Minimal Xenon Poisoning*, Nucl. Sci. Eng., 6, No. 2, 152-156 (Aug. 1959).

M. Ash, *The Xenon Minimax Problem*, IA-988 (Sept. 1964).

M. Ash, *Application of Dynamic Programming to Optimal Shutdown Control*, Nucl. Sci. Eng., 24, No. 1, 77-86 (Jan. 1966).

M. Ash, *Optimal Shutdown Control of Nuclear Reactors*, (New York: Academic Press, 1966).

D. Babala, *Interval Distributions in Neutron Counting Statistics*, Nucl. Sci. Eng., 28, No. 2, 243-246 (May 1967).

D. Babala, *Point-Reactor Theory of Rossi-Alpha Experiment*, Nucl. Sci. Eng., 28, No. 2, 237-242 (May 1967).

SELECTED NUCLEAR BIBLIOGRAPHY

D. Babala, *Neutron Counting Statistics in Nuclear Reactors*, KR-114 (Nov. 1966).

D. Babala, *On the Theory of Rossi-Alpha Experiment in Reactor Noise Studies*, Nucl. Sci. Eng., 24, No. 3, 418-424 (Nov. 1966).

S. J. Ball and R. K. Adams, *MATEXP, A General Purpose Digital Computer Program for Solving Ordinary Differential Equations by the Matrix Exponential Method*, ORNL-TM-1933 (Aug. 1967).

W. Baran and K. Meyer, *Effect of Delayed Neutrons on the Stability of a Nuclear Power Reactor*, Nucl. Sci. Eng., 24, No. 4, 356-361 (Apr. 1966).

P. R. Barrett and J. J. Thompson, *Stochastic Approach to the Equation of Power Reactor Kinetics*, Energ. Nucl. (Milan), 15, 461-472 (July 1968).

N. H. Barth, *Physics Calculations for the Consumers Big Rock Point Process Computer*, GEAP-3932 (May 1963).

A. S. Bartu, *Use of On-line Computers in Boiling-Water-Reactor Power Plants*, Neue Tech., 9, No. A3, 146-151 (May 1967).

A. Batenburg, *Controller Optimization for Pressurized Water Reactor*, Ph.D. Thesis, Univ. Pittsburgh, 1967. (UM No. 68-9735)

W. C. Bean, *Feedback System Concepts*, WAPD-TM-724 (Feb. 1968).

E. Beckjord, *Nuclear Performance Computer for Big Rock Point*, Nucleonics, 20, No. 6, 57-58 (June 1962).

L. Beltracchi, *Stability Criteria for a Nonlinear Reactor Control System*, IEEE Trans. Nucl. Sci., NS-10, No. 4, 30-46 (Sept. 1963).

L. Beltracchi, *An Analysis of a Nonlinear Reactor Control System*, IEEE Trans. Nucl. Sci., NS-12, No. 4, 323-342 (Aug. 1965).

A. Benmergi, *The Design of a Program for Analog Adjustment Using a Digital Computer at Saint-Laurent-Des-Eaux*, Energ. Nucl. (Paris), 8, No. 6, 411-417 (Sept. - Oct. 1966). (In French)

E. F. Bennett, *The Rice Formulation of Pile Noise*, Nucl. Sci. Eng., 8, No. 1, 53-61 (July 1960).

G. Birkhoff, *Numerical Solution of the Reactor Kinetics Equations*, Numerical Solution of Nonlinear Differential Equations, (ed.) D. Greenspan (New York: John Wiley & Sons, Inc., 1966), 3-20.

R. A. Blaine and R. F. Berland, *Simulation of Reactor Dynamics. Vol. I. A Description of Airos II-A*, NAA-SR-12452 (Sept. 1967).

SELECTED NUCLEAR BIBLIOGRAPHY

H. Borgwaldt and D. Stegemann, *A Common Theory for Neutronic Noise Analysis Experiments in Nuclear Reactors*, Nukleonik, 7, No. 6, 313-325 (July 1965).

H. Borgwaldt, *Neutron Noise in a Reactor with an External Control Loop*, Nukleonik, 11, No. 2, 76-84 (May 1968).

G. S. Brunson et al., *Measuring the Prompt Period of a Reactor*, Nucleonics, 15, No. 11, 132-141 (Nov. 1957).

J. B. Bullock, *Computer Control for Reactors*, Nucl. Safety, 8, No. 2, 138-139 (Winter 1966-1967).

T. R. Bump and H. O. Monson, *Predicted Dynamic Behavior of EBR-II*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 11, 404-419.

W. L. Bunch, L. D. Philipp, and M. R. Wood, *Time-to-Power, A Novel Startup Parameter*, Nucl. Appl., 2, No. 5, 357-362 (Oct. 1966).

G. Burnand, *Application of Liapunov's Second Method to the Study of the Stability of Nuclear Reactors*, EIR-125 (Nov. 1967). (In French)

R. H. Campbell and R. N. H. McMillan, *The Control Characteristics of the Various Types of Nuclear Plants*, Proc. IEE Symp. Automat. Contr. Elect. Supply (Manchester, England, Mar. 1966), 229-254.

J. Canosa, *A New Method for Nonlinear Reactor Dynamics Problems*, Nukleonik, 9, No. 6, 289-295 (Apr. 1967).

J. Chernick, *A Review of Nonlinear Reactor Dynamics Problems*, BNL-6302 (July 1962).

W. Ciechanowicz, *On Transient Digital Control of Large Nuclear Power Reactors*, Nucl. Sci. Eng., 31, No. 3, 465-473 (Mar. 1968).

W. Ciechanowicz and S. Bogumil, *On the On-line Statistical Identification of Nuclear Power Reactor Dynamics*, Nucl. Sci. Eng., 31, No. 3, 474-483 (Mar. 1968).

R. G. Clark, *Operation of Reactor Systems with a Digital Computer*, Trans. Amer. Nucl. Soc., 11, No. 1, 339-340 (June 1968).

W. G. Clarke et al., *Variances and Covariances of Neutron and Precursor Populations in Time-Varying Reactors*, Trans. Amer. Nucl. Soc., 9, No. 1, 119-120 (June 1966).

W. G. Clarke et al., *Variances and Covariances of Neutron and Precursor Populations in Time-Varying Reactors*, Nucl. Sci. Eng., 31, No. 3, 440-457 (Mar. 1968).

SELECTED NUCLEAR BIBLIOGRAPHY

L. D. Coffin et al., *Computer Programs for Operation of the High Temperature Lattice Test Reactor (HTLTR), Vol. I,* BNWL-651 (Dec. 1967).

C. E. Cohn, *A Simplified Theory of Pile Noise,* Nucl. Sci. Eng., 7, No. 5, 472-475 (May 1960).

C. E. Cohn, *Initial Usage of a Computer for On-line Data Reduction in Reactor Physics Experiments,* Trans. Amer. Nucl. Soc., 8, No. 2, 585 (Nov. 1965).

C. E. Cohn, *Further Use of an On-line Computer in Reactor Physics Experiments,* Trans. Amer. Nucl. Soc., 9, No. 1, 262 (June 1966).

C. E. Cohn, *Two Advances in Reactor Neutron Noise Analysis,* Power Reactor Technol., 9, No. 3, 142-144 (Summer 1966).

C. E. Cohn, *Automated Data Analysis and Control for Critical Facilities,* Use of Computers in Analysis of Experimental Data and the Control of Nuclear Facilities, CONF-660527 (May 1967), 49-66.

G. B. Collins and A. Hopkinson, *Dynamic Model for the Prototype Fast Reactor Steam Generator and Steam Plant,* AEEW-R420 (1965).

G. B. Collins, *A Survey of Digital Instrumentation and Computer Interface Methods and Developments,* AEEW-R595 (June 1968).

M. Combet, J. Eder, H. H. van Zonneveld, *Digital Periodmeter for a Reactor,* L'Onde Electrique, 46, No. 466, 1244-1250 (Jan. 1966). (In French)

E. D. Courant and P. R. Wallace, *Fluctuations of the Number of Neutrons in a Pile,* Phys. Rev., 72, No. 11, 1038-1048 (Dec. 1947).

R. T. Cox and J. Walker, *Control of Nuclear Reactors,* Proc. IEE, Pt. B, 103, No. 11, 577-589 (Sept. 1956).

R. L. Crowther and L. K. Holland, *Nuclear Applications of On-line Process Computers,* Nuclear News, 11, No. 5, 52-56 (May 1968).

J. D. Cummins, *Identification and Optimization Studies of the Dynamics of a Small Experimental Boiler,* AEEW-R580 (Oct. 1967).

A. Dalfes, *A Study of Stochastic Kinetics of Nuclear Reactors,* Nukleonik, 4, No. 7, 299-303 (Oct. 1962).

A. Dalfes, *The Fokker-Planck and Langevin Equations of a Nuclear Reactor,* Nukleonik, 5, No. 8, 348-352 (Dec. 1963).

A. Dalfes, *The Correlation Function and Power Spectral Density of Nuclear Reactors,* Nukleonik, 6, No. 2, 53-58 (Mar. 1964).

SELECTED NUCLEAR BIBLIOGRAPHY

A. Dalfes, *Some Considerations on Nuclear Reactor Noise*, Nukleonik, 7, No. 7, 426-427 (Aug. 1965).

A. Dalfes, *The Random Processes of a Nuclear Reactor and Their Detection*, Nukleonik, 8, No. 2, 94-101 (Feb. 1966).

A. Dalfes, *Concept of Transfer Functions for a Nuclear Reactor*, CEA-R-3102 (Dec. 1966). (In French)

A. Dalfes, *Functional Analysis of the Random Processes in Nuclear Reactors*, Nukleonik, 9, No. 3, 123-129 (Feb. 1967).

R. S. Darke and H. H. Stevens, *Application of Fossil Plant Experience with On-line Computers*, Trans. Amer. Nucl. Soc., 9, No. 1, 266 (June 1966).

H. L. Davis, *Looking Ahead to Nuclear-Plant Automation*, Nucleonics, 20, No. 6, 52-54 (June 1962).

T. F. Davis et al., *Bibliographies of Interest to the Atomic Energy Program 1962 through 1966*, TID-3350 (Aug. 1968).

W. DeBacker, *Synthesis of Optimal Control and Hybrid Computation*, Proc. 4th Int. Anal. Comput. Meet. (Brighton, England, Sept. 1964), 265-267.

W. DeBacker, *Reactor Power Reduction in Minimum Time Including Xenon Poisoning*, EUR-2056.E(June 1965).

F. DeHoffman, *Intensity Fluctuations of a Neutron Chain Reactor*, MDDC 382 (Oct. 1946).

F. DeHoffman, *Statistical Aspects of Pile Theory*, Science and Engineering of Nuclear Power, 2, (Reading, Mass.: Addison-Wesley Publishing Co., 1949), 103-119.

W. A. Dempler, *Second Derivative of Flux Digital Control of a Space Reactor from Source Level to Critical*, IEEE Trans. Nucl. Sci., NS-12, No. 1, 117-125 (Feb. 1965).

H. B. Demuth, F. P. Shilling, and L. Weintraub, *Ultra High Temperature Experiment On-line Process Computer*, Neutron Dynamics and Control, CONF-650413 (May 1966), 518-530.

H. B. Demuth et al., *Digital Control System for the UHTREX Reactor*, LA-DC-9638 (Sept. 1968).

J. B. Dragt, *Accurate Reactor Noise Measurements in a Low Power Critical Reactor*, Nukleonik, 8, No. 4, 188-193 (May 1966).

J. B. Dragt, *Reactor Noise Analysis by Means of Polarity Correlation*, Nukleonik, 8, No. 4, 225-226 (May 1966).

SELECTED NUCLEAR BIBLIOGRAPHY

J. B. Dragt and E. Turkcan, *Some Remarks on the Practical Use of the P-Method in Reactor Noise Analysis*, Nukleonik, 10, No. 2, 67-69 (July 1967).

J. B. Dragt, *Bias due to Finite Measuring Time in Nuclear Reactor Noise Analysis*, RCN-64 (1967).

M. Edelmann, T. E. Murley, D. Stegemann, *Investigation of Prompt Neutron Kinetics in the Fast-Thermal Argonaut Reactor (Stark) by Noise Analysis*, KFK-522 (Jan. 1967).

R. A. Edwards, *Digital Monitors for Nuclear Plants*, Nucleonics, 20, No. 6, 59-64 (June 1962).

J. R. Engel, *Application of an On-line Digital Computer to a Reactor Experiment*, Trans. Amer. Nucl. Soc., 8, No. 2, 585-586 (Nov. 1965).

W. K. Ergen, H. J. Lipkin, and J. A. Nohel, *Applications of Liapunov's Second Method in Reactor Dynamics*, J. Math. Phys., 36, 36-48 (1957).

P. B. F. Evans, *Control and Instrumentation of Prototype Fast Reactor*, Proc. Conf. Fast Breeder Reactors (London, England, May, 1966).

Feasibility Study of Digital Computer Control of a Nuclear Power Plant, IBM Rep. No. 60-511-10 (June 1960).

R. Feynman, F. DeHoffman, and R. Serber, *Dispersion of the Neutron Emission in U-235 Fission*, J. Nucl. Energy, 3, No. 1/2, 64-69 (1956).

R. P. de Figueiredo, *On the Nonlinear Stability of a Nuclear Reactor with Delayed Neutrons*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 11, 237-244.

C. D. Flowers and L. H. Gerhardstein, *Analog-Hybrid Dynamic Simulation of the FFTF Reactor and Heat Transport System*, BNWL-707 (Apr. 1968).

Ford Instrument Co., *Application of Digital Techniques to Reactor Control Systems*, NYO-8502 (Oct. 1957).

Ford Instrument Co., *Digital Start-up Control for Aircraft Reactors*, NYO-8586 (Mar. 1958).

J. Freycenon, *Harmonic Analysis of the Instantaneous Counting Rate of a Periodically Sampled Flow of Pulses*, Noise Analysis in Nuclear Systems, TID-7679 (June 1964), 331-334.

J. Freycenon, *Correlation between Successive Counts at the Output of a Numerical Divider Driven by a Train of Poisson Impulses*, L'Onde Electrique, 46, No. 466, 1231-1236 (Jan. 1966). (In French)

SELECTED NUCLEAR BIBLIOGRAPHY

J. Freycenon, *Theoretical Study of a Random Process Derived from a Poisson Process by Division*, CEA-R 2907 (Mar. 1966). (In French)

C. L. Fruhauf, *RKC Reactor Kinetics Calculations Program*, KAPL-M-SR-1 (Aug. 1966).

J. Fuan, J. Furet, and J. Kaiser, *The Use of Digital Techniques for Nuclear Instrumentation*, IEEE Trans. Nucl. Sci., NS-14, No. 1, 233-240 (Feb. 1967).

T. Fujisawa and K. Sumita, *Automatic Start-up of a Research Reactor, I: Stability Analysis*, J. Nucl. Sci. Technol., 1, No. 7, 264-275 (Oct. 1964).

T. Fujisawa, K. Watanabe, and K. Sumita, *Automatic Start-up of a Research Reactor; II: Design of a LCRPM and Automatic Start-up Experiments*, J. Nucl. Sci. Technol., 1, No. 9, 350-361 (Dec. 1964).

T. Fujisawa, H. Koshii, and K. Watanabe, *Computer Control of the Start-up of a Nuclear Reactor*, JAERI 1152 (Aug. 1967). (In Japanese)

J. Furet, *Digital Techniques in the Control of Atomic Piles*, CEA-2175 (1962). (In French)

J. Furet, B. Jacquemin, and J. Kaiser, *Numerical Techniques in the Control of Nuclear Reactors*, L'Onde Electrique, 44, No. 448-449, 758-768 (July - Aug. 1964). (In French)

J. Furet and J. Weill, *Fast Start of Atomic Reactors*, L'Onde Electrique, 44, No. 448-449, 812-815 (July - Aug. 1964). (In French)

J. Furet, *Safety Period during the Start-up of Nuclear Reactors*, ORNL-TR-292 (Nov. 1964).

J. Furet and J. Pupponi, *Some Special Aspects of Control in Nuclear Power Reactors*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 213-219.

J. Furet, *Control and Instrumentation of Nuclear Reactors*, (Paris: Masson Et Cie, Editeurs, 1968). (In French)

S. J. Gage, F. T. Adler, and P. N. Powers, *Investigations on Nonlinear Stability of Coupled Nuclear Systems*, Neutron Dynamics and Control, CONF-650413 (May 1966), 45-76.

S. J. Gage, *Investigations on the Dynamic Behavior of Couple-Core Nuclear Reactors*, Ph.D. Thesis, Purdue Univ., 1966. (UM No. 66-7421)

W. M. Gaines and T. J. Glass, *What We Can Learn from Conventional-Plant Automation*, Nucleonics, 20, No. 6, 69-70 (June 1962).

SELECTED NUCLEAR BIBLIOGRAPHY

D. P. Gelopulos, *Computation of Regions of Constrained Stability for Nonlinear Control Systems*, Ph.D. Thesis, Univ. Arizona, 1967. (UM No. 68-785)

L. H. Gerhardstein, *Model Development Considerations in the Simulation of Fast Nuclear Power Plants*, BNWL-SA-1443 (Oct. 1967).

D. Gertz and J. Levine, *Feasibility Study of a Unified Digital Instrumentation and Control System for Nuclear Power Plants*, NYO-2764-2 (June 1965).

K. L. Gimmy and F. R. Field, *On-line Computer Assists Reactor Operation at Savannah River Plant*, Trans. Amer. Nucl. Soc., Supp. to Vol. 8, 53-54 (July 1965).

P. Giordano, A. Mathis, and G. Scandellari, *The Design of a Nonlinear Reactor Control System*, IEEE Trans. Nucl. Sci., NS-12, No. 2, 2-11 (April 1965).

R. S. Gow and D. J. Millard, *Automatic Control in Gas-Cooled-Reactor Nuclear Power Stations*, Proc. IEE Symp. Auto. Control Elect. Supply, (Manchester, England, Mar. 1966), 255-283.

P. G. Greene, *Direct Digital Control of a Nuclear Reactor*, Instrum. Contr. Syst., 38, No. 6, 85-88 (June 1965).

E. F. Groh and C. E. Cohn, *A Simple Compact Rod Drive Using a Stepping Motor*, Nucl. Sci. Eng., 20, No. 3, 290-297 (Nov. 1964).

C. Grunberger and M. Morin, *Data Handling at the Chinon Nuclear Center: EDF 3*, Energ. Nucl. (Paris), 6, No. 3, 135-141 (May 1964). (In French)

E. P. Gyftopoulos and P. M. Coble, *A Digital Nuclear Reactor Control System*, AIEE Trans. Appl. Ind., 79, No. 51, 305-314 (Nov. 1960).

E. P. Gyftopoulos, *Lagrange Stability by Liapunov's Direct Method*, Reactor Kinetics and Control, TID-7662 (Apr. 1964), 227-237.

E. P. Gyftopoulos, *Nonlinear Reactor Kinetics*, MMPP-FRPC-64-3 (June 1964).

E. P. Gyftopoulos, *Point Reactor Kinetics and Stability Criteria*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 3-11.

E. P. Gyftopoulos and M. Green, *Experimental Interpretation of a Criterion on Nonlinear Stability*, Trans. Amer. Nucl. Soc., 8, No. 2, 476-477 (Nov. 1965).

E. P. Gyftopoulos, *Theoretical and Experimental Criteria for Nonlinear Reactor Stability*, Nucl. Sci. Eng., 26, No. 1, 26-33 (Sept. 1966).

SELECTED NUCLEAR BIBLIOGRAPHY

L. J. Habegger and R. E. Bailey, *The Use of a Kalman Filter for the Determination of EBWR Dynamic Characteristics*, Trans. Amer. Nucl. Soc., 11, No. 1, 237-238 (June 1968).

K. F. Hansen and P. K. Koch, *GAPOTKIN: A Point Kinetics Code for the UNIVAC 1108*, GA-8204 (Oct. 1967).

D. R. Harris, *The Sampling Estimate of the Parameter Variance/Mean in Reactor Fluctuation Measurements*, WAPD-TM-157 (Aug. 1958).

D. R. Harris, *Stochastic Fluctuations in a Power Reactor*, WAPD-TM-190 (Nov. 1958).

D. R. Harris, *Kinetics of Low Source Level*, Naval Reactors Physics Handbook, 1, TID-7030 (1964), 1010-1130.

D. R. Harris, *Neutron Fluctuations in a Reactor of Finite Size*, Nucl. Sci. Eng., 21, No. 3, 369-381 (Mar. 1965).

D. R. Harris and V. Prescop, *Stability and Stationarity of a Reactor as a Stochastic Process with Feedback*, WAPD-T-1896 (Mar. 1968).

H. Hejtmanek, *Time Behavior of a Reactor and Ergodic Theory of Semigroups*, J. Math. Phys., 8, No. 7, 1401-1405 (July 1967).

H. Hejtmanek, *Ergodic Theory and Discrete Transport Processes*, AD-664026 (Oct. 1967).

J. B. Henshall, E. P. Elkins, and W. F. Carlson, *Use of an On-line Computer in the Testing of Prototype Nuclear Rocket Engines*, LA-DC-9460 (June 1968).

R. J. Hermsen and T. J. Higgins, *Optimal Design of Nuclear Reactor Control Systems by Use of Pontryagin's Maximum Principle*, Proc. IEEE Region 6 Annu. Conf. (Salt Lake City, April, 1964).

K. G. Hilton, *Some Methods of Automatic Start-up of Power Station Plant*, Proc. IEE Symp. Auto. Control Elect. Supply (Manchester, England, Mar. 1966), 46-62.

D. S. Hiorns and M. W. Jervis, *The Use of Computers in the Control and Instrumentation of Central Electricity Generating Board Nuclear Power Stations*, J. Brit. Nucl. Energy Soc., 5, No. 1, 103-113 (Jan. 1966).

T. J. Hirons, *Approximate Analytical Solutions of the Reactor Kinetics Equations*, Ph.D. Thesis, North Carolina State Univ., 1966. (UM No. 67-7405)

G. L. Hohmann and B. G. Strait, *Automatic Startup for Nuclear Reactors*, LA(MS)-3111 (Aug. 1964).

SELECTED NUCLEAR BIBLIOGRAPHY

R. L. Holladay, *The Functions and Performance of the Consumers Big Rock Point Process Computer System*, GEAP-5297 (Oct. 1966).

L. K. Holland, *On-line Computer Experience with Boiling-Water Reactors*, Trans. Amer. Nucl. Soc., 9, No. 1, 264 (June 1966).

A. Hopkinson, *Two Models for the Dynamics of a Cross Flow Heat Exchanger*, AEEW-R258 (Dec. 1962).

A. Hopkinson, *Prototype Fast Reactor Primary Circuit and Intermediate Heat Exchanger Dynamic Model*, AEEW-R418 (Feb. 1965).

A. Hopkinson, *A Core Outlet Temperature Controller for the Prototype Fast Reactor*, AEEW-R474 (Mar. 1966).

A. Hopkinson and R. W. Levell, *Analogue and Digital Computing Techniques in Plant Modelling, Reduction and Control Studies*, Advances in Computer Control (IEE Conf. Pub. No. 29), Paper No. C10 (Apr. 1967).

C. Hsu and R. E. Bailey, *Optimal Control of Spatially Dependent Nuclear Reactors*, Trans. Amer. Nucl. Soc., 10, No. 1, 253 (June 1967).

C. Hsu, *Control and Stability Analysis of Spatially-Dependent Nuclear Reactor Systems*, Ph.D. Thesis, Purdue Univ., 1967, (UM No. 67-16,655); published as ANL-7322 (July 1967).

K. A. Hub et al., *Feasibility Study of Nuclear Steam Supply System Using 10,000-MW, Sodium-Cooled Breeder Reactor*, ANL-7183 (Sept. 1966).

T. J. Hurley, Jr., *Axial Power Monitoring with an On-line Computer*, Trans. Amer. Nucl. Soc.,9, No. 1, 262-263 (June 1966).

H. Hurwitz et al., *Kinetics of Low Source Reactor Startups*, TID-16142 (June 1962).

H. Hurwitz et al., *Kinetics of Low Source Reactor Startups, Part I*, Nucl. Sci. Eng., 15, No. 2, 166-186 (Feb. 1963).

H. Hurwitz et al., *Kinetics of Low Source Reactor Startups, Part II*, Nucl. Sci. Eng., 15, No. 2, 187-196 (Feb. 1963).

J. C. Jacquin, G. Martinot, and J. P. Satre, *General Organization of the Control of the EL-4 Nuclear Reactor*, Measures Regulation Automatisme, 31, No. 1, 80-86 (Jan. 1966). (In French)

R. L. Johnson, *A Statistical Determination of the Reduced Prompt Neutron Generation Time in the Spert IV Reactor*, IDO-16903 (Aug. 1963).

K. Kadavanich, *The Identification of Nuclear Reactor Parameters Through Spaced Reactivity Insertions*, Ph.D. Thesis, Purdue Univ., 1967. (UM No. 67-10,215)

SELECTED NUCLEAR BIBLIOGRAPHY

T. Kagayama, *Dynamic Analysis in Start-up of a Nuclear Reactor*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 11, 317-322.

G. R. Keepin and C. W. Cox, *General Solution of the Reactor Kinetic Equations*, Nucl. Sci. Eng., 8, No. 6, 670-690 (Dec. 1960).

L. G. Kemeny and W. Murgatroyd, *Stochastic Models for Fission Reactors*, Noise Analysis in Nuclear Systems, TID-7679 (June 1964), 29-59.

L. G. Kemeny, *Fundamental Aspects of Stochastic Processes and Fluctuation Phenomena in Fission Reactors*, Neutron Noise, Waves, and Pulse Propagation, CONF-660206 (May 1967), 531-566.

H. K. Kim, *Sensitivity Analysis of Reactor Power Kinetics*, Ph.D. Thesis, Univ. North Carolina, 1967, (UM No. 67-11984)

I. Kliger, *Optimal Control of Space Dependent Nuclear Reactors*, Trans. Amer. Nucl. Soc., 8, No. 1, 233-234 (June 1965).

J. R. Kosorok, *Interim Report on Control Program Development*, BNWL-348 (Dec. 1966).

P. Kovanic, *Optimum Digital Information Treatment and Nuclear Engineering Instrumentation*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 235-242.

P. Kovanic and Y. Rygl, *Digital Follow-Up System for Nuclear Engineering*, ANL-TRANS-469 (Apr. 1967).

B. Kozik, *Statistical Basis of the Application of the Dynamical Model for Stationary Nuclear Reactors*, ANL-TRANS-415 (December 1966). Translation of At. Energ. (USSR), 20, No. 1, 21-26 (Jan. 1966).

S. H. Kyong and E. P. Gyftopoulos, *Input-Output Approach to Optimal Control*, Trans. Amer. Nucl. Soc., 8, No. 2, 479-480 (Nov. 1965).

B. Larssen, *The Use of Statistical Methods in the Determination of Reactor Dynamics*, Neue Tech. 9, No. A3, 141-145 (May 1967).

B. R. Lawrence, *Neutron Density Fluctuations Induced by Hydraulic Noise in a Nuclear Power Reactor*, ORNL-TM-1332.

B. R. Lawrence and E. P. Epler, *Further Use of On-line Digital Computers in the Operation of Power Reactors*, Nucl. Safety, 7, No. 4, 456-458 (Summer 1966).

B. R. Lawrence, *Determination of the Power vs. Reactivity Frequency Response Function of a Power Reactor, with Application to the High Flux Isotope Reactor*, ORNL-TM-1471 (July 1966).

SELECTED NUCLEAR BIBLIOGRAPHY

B. R. Lawrence, H. P. Danforth, and J. B. Bullock, *A Mathematical Model for the High Flux Isotope Reactor Reactivity Calculation*, ORNL-TM-1472 (Oct. 1966).

L. A. J. Lawrence and A. B. Keats, *Digital Methods Applied to Redundant Auto-Controllers for Plant*, AEEW-M603 (Dec. 1965).

S. N. Lehr and V. P. Mathis, *A Digital Startup Control for Mobile Nuclear Reactors*, Proc. Nucl. Sci. Eng. Conf. (Cleveland, Ohio, April 1959).

S. N. Lehr and V. P. Mathis, *A Digital Start-up Control for Air-Cooled Nuclear Reactors*, AIEE Trans., Part I, 79, No. 3, 369-375 (Sept. 1960).

G. S. Lellouche, *Reactor-Kinetics Stability Criteria*, Nucl. Sci. Eng., 24, No. 1, 72-76 (Jan. 1966).

C. G. Lennox and A. Pearson, *Thermal Power Control of the NRU Reactor*, IRE Trans. Nucl. Sci., NS-5, No. 2, 68-72 (Aug. 1958).

C. G. Lennox and A. Pearson, *NRU Reactor Neutron Level Control System*, IRE Trans. Nucl. Sci., NS-5, No. 2, 64-67 (Aug. 1958).

C. G. Lennox and A. E. Pearson, *High-Speed Monitor for Closed-Loop Control*, Nucleonics, 20, No. 6, 73-74 (June 1968).

C. G. Lennox and N. P. Vakil, *Digital Computer Control Experiment on Nuclear Reactor*, Proc. 4th Conf. Computing and Data Processing Society of Canada (May 1964), 46-58.

B. E. Lenord, *Dynamic Reactivity in Nuclear Reactors*, Proc. JACC, 198-199 (1967).

H. R. Leribaux, *Stochastic Processes in Coupled Nuclear Reactor Cores*, Ph.D. Thesis, Iowa State Univ., 1963. (UM No. 63-7259)

R. W. Levell, *Computer Studies of the Prototype Fast Reactor Uncontrolled Primary Circuit Dynamic Model*, AEEW-R419 (1965).

R. W. Levell and A. Hopkinson, *Simplification of the Prototype Fast Reactor Dynamic Model*, AEEW-R422 (Sept. 1965).

J. Lewins, *The Use of the Generation Time in Reactor Kinetics*, Nucl. Sci. Eng., 7, No. 2, 122-126 (Feb. 1960).

J. Lewins and R. D. Benham, *Time Optimal Xenon Shutdown on the Xenon Boundary*, BNWL-186 (May 1966).

J. Lewins and A. L. Babb, *Optimum Nuclear Reactor Control Theory*, Advances in Nucl. Sci. Tech., 4, (eds.) P. Greebler and E. J. Henley (New York: Academic Press, 1968), 251-308.

SELECTED NUCLEAR BIBLIOGRAPHY

J. Lewins et al., *Energy Optimal Xenon Shutdown*, Nucl. Sci. Eng., 31, No. 2, 272-281 (Feb. 1968).

J. G. Lewis, L. K. Holland, and R. L. Holladay, *On-Line Computer at Big Rock Point*, Nucleonics, 22, No. 11, 46-47 (Nov. 1964).

A. J. Lindeman and L. Ruby, *Subcritical Reactivity from Neutron Statistics*, Nucl. Sci. Eng., 29, No. 2, 308-310 (Aug. 1967).

W. W. Little, Jr., *Kinetics I, A Neutron Kinetics Code in FORTRAN IV*, BNWL-111 (July 1965).

T. C. Liu, *Optimum Control of Nuclear Rockets Using Models with Distributed Parameters*, Ph.D. Thesis, Rennselear Polytechnic Institute, 1966. (UM No. 66-9346)

R. M. Lord, *A Kinetics Study of a 600 MW(H) Prototype Fast Reactor*, TRG Report 1448(R) (Feb. 1967).

J. E. Lunde, *Computer Control of the HBWR*, Atoomenergie Haar Toepass., 9, No. 11, 267-273 (Dec. 1967).

D. R. MacFarlane, *An Analytic Study of the Transient Boiling of Sodium in Reactor Coolant Channels*, ANL-7222 (June 1966).

D. B. MacMillan, *Asymptotic Numerical Solution of the Reactor Kinetics Equations*, KAPL-M-6581 (Nov. 1966).

D. B. MacMillan, *Asymptotic Methods for Systems of Differential Equations in Which Some Variables Have Very Short Response Times*, KAPL-P-3341 (May 1967).

D. B. MacMillan, *Asymptotic Formulas for Initial-Value Problems of Non-linear Differential Equations with a Small Parameter Multiplying the Derivative*, KAPL-3359 (June 1967).

G. Marcillat, *Automatic Control of Nuclear Reactors*, L'Onde Electrique, 46, No. 466, 1237-1243 (Jan. 1966). (In French)

T. J. Marciniak, *Comments on "Stability Analysis of a Sampled-Data Controlled Nuclear Reactor System,"* Nucl. Sci. Eng., 28, No. 2, 315-316 (May 1967).

T. J. Marciniak, *Study of a Linearized Sampled-Data Control Algorithm for Zero-Power Reactors*, Trans. Amer. Nucl. Soc., 10, No. 1, 312 (June 1967).

J. Martin, *Development of a Power-Period Calculation Unit for Nuclear Reactor Control*, CEA-R-3026 (1966).

SELECTED NUCLEAR BIBLIOGRAPHY

G. Martinot, J. C. Jacquin, and C. Leroy, *Instrumentation and Control of the EL4 Reactor*, Proc. Symp. Heavy-Water Power Reactors (Vienna, Austria, Sept. 1967), 905-924. (In French)

W. Matthes, *Statistical Fluctuations and Their Correlations in Reactor Neutron Distributions*, Nukleonik, 4, No. 5, 213-226 (July 1962).

W. Matthes, *Measurement of the Transfer Function with Statistical Methods*, Nukleonik, 8, No. 1, 21-32 (Jan. 1966).

W. Matthes, *Theory of Fluctuations in Neutron Fields*, Nukleonik, 8, No. 2, 87-94 (Feb. 1966).

W. Matthes, *Noise Analysis of Periodically Pulsed Reactors*, Nukleonik, 8, No. 6, 329-333 (Aug. 1966).

E. J. McGrath, *Statistical Analysis of Estimates for the Power Spectral Density in Neutron Multiplying Systems*, Ph.D. Thesis, Univ. Washington, 1966. (UM No. 66-7885)

E. J. McGrath and R. W. Albrecht, *Variance Analysis of Estimates for the Power-Spectral Density of Neutron Multiplying Systems*, Trans. Amer. Nucl. Soc., 9, No. 1, 121-122 (June 1966).

A. Medina, *Stochastic Models for Nuclear Reactor*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 16, 697-700.

J. Megy et al., *Data Handling in Nuclear Reactors*, Bull. Inform. Sci. Tech., No. 61, 1-28 (May 1962). (In French)

J. Miida, *Kinetic Studies and Control of Reactors by Statistical Methods*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 151-163.

A. I. Mogil'ner and V. G. Zolotukhin, *Measuring the Characteristics of Kinetics of a Reactor by the Statistical p-Method*, At. Energ. (USSR), 10, No. 4, 365-367 (Apr. 1961).

R. R. Mohler, *Stability and Control of Nuclear Rocket Propulsion*, IRE Trans. Automat. Contr., AC-7, No. 2, 86-96 (Mar. 1962).

R. R. Mohler, *Let's Apply Optimal Control Theory and Nonlinear Analysis*, Power Reactor Technol., 9, No. 4, 169-173 (Fall 1966).

M. N. Moore, *The Determination of Reactor Transfer Functions from Measurements at Steady Operation*, Nucl. Sci. Eng., 3, No. 4, 387-394 (Apr. 1958).

K. R. Morin, *The NRU Monitor Computer and Self-Powered Neutron Flux Detectors*, AECL-2216 (May 1965).

SELECTED NUCLEAR BIBLIOGRAPHY

R. Morin, *Use of a Digital Computer for the Start-up and Operation of EDF3*, New Techniques, 8, No. A1, 26-32 (Jan. 1966). (In French)

R. Morin, *Utilization of Digital Computers for Starting and Running the EDF-3 Atomic Power Plant*, AEC-TR-691 (Dec. 1967).

H. S. Murray and L. E. Weaver, *Stability of Coupled-Core Nuclear Reactor Systems*, NASA-CR-447 (June 1965).

H. S. Murray, *The Stability of Coupled-Core Nuclear Reactor Systems by the Second Method of Liapunov*, Ph.D. Thesis, Univ. Arizona, 1965. (UM No. 65-11558)

H. S. Murray, D. G. Schultz, and L. E. Weaver, *Stability of Coupled Core Reactors by the Second Method of Liapunov*, Reactor Sci. Technol., 20, 729-734 (Sept. 1966).

H. S. Murray, *State-Variable Feedback Control of Coupled Nuclear Systems*, LA-DC-8352 (1967).

M. Natelson, R. K. Osborn, and F. Shure, *Space and Energy Effects in Reactor Fluctuation Experiments*, Reactor Sci. Technol. 20, 557-585 (July 1966).

T. Nomura, S. Gotoh, and K. Yamaki, *Reactivity Measurements by Neutron Noise Analysis Using Two-Detector Correlation Method and Supercritical Reactor Noise Analysis*, Neutron Noise, Waves, and Pulse Propagation, CONF-660206 (May 1967), 217-246.

L. D. Noble, *A Nonlinear Analysis of Reactors with Arbitrary Linear Feedback*, Ph.D. Thesis, Univ. Michigan, 1965. (UM No. 66-6666)

L. W. Nordheim, *Pile Kinetics*, MDDC-35 (June 1946).

J. D. Orndoff, *Prompt Neutron Periods of Metal Critical Assemblies*, Nucl. Sci. Eng., 2, No. 4, 450-460 (July 1967).

R. K. Osborn and S. Yip, *Physical Theory of Neutron Noise in Reactors and Reactor-Like Systems*, Noise Analysis in Nuclear Systems, TID-7679 (June 1964), 1-12.

R. K. Osborn, *Speculations on the Interpretation of Neutron Noise Experiments*, ORNL-3757 (Jan. 1965).

R. K. Osborn and M. Natelson, *Kinetic Equations for Neutron Distributions*, Reactor Sci. Technol., 19, No. 8, 619-639 (1965).

R. K. Osborn and A. Z. Akcasu, *Some Theorems on Neutron Fluctuations and Fluctuation Spectra*, Neutron Dynamics and Control, CONF-650413 (May 1966), 531-543.

SELECTED NUCLEAR BIBLIOGRAPHY

R. K. Osborn and J. M. Nieto, *Detector Effects on the Statistics of Nuetron Fluctuations*, Nucl. Sci. Eng., 26, No. 4, 511-516 (Dec. 1966).

L. I. Pal, *Statistical Fluctuations of Neutron Multiplication*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 16, 687-696.

L. Pal, *Statistical Theory of Neutron Chain Reactions*, Acta Phys. Hung., 14, 345-380 (1962). (In Russian)

L. Pal, *Determination of the Prompt Neutron Period from the Fluctuations of the Number of Neutrons in a Reactor*, Reactor Sci. Technol., 17, No. 9, 395-409 (1963).

A. Pearson, *The Future of the Digital Computer in Power Reactor Instrumentation*, Trans. Amer. Nucl. Soc., 9, No. 1, 266-267 (June 1966).

A. Pearson, *The NRU Computer-Control Experiment*, Use of Computers in Analysis of Experimental Data and the Control of Nuclear Facilities, CONF-660527 (May 1967), 21-32.

P. R. Pluta, *An Analysis of Nuclear Reactor Fluctuations by Methods of Stochastic Processes*, Ph.D. Thesis, Univ. Michigan, 1962, (UM No. 63-432)

P. R. Pluta, *Probabilistic Analysis of Reactor Kinetics*, Reactor Kinetics and Control, TID-7662 (Apr. 1964), 136-149.

V. M. Popov, *Notes on the Inherent Stability of Nuclear Reactors*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 11, 245-250.

V. M. Popov, *A New Criterion Regarding the Stability of Systems Containing Nuclear Reactors*, BNL-TR-38 (Dec. 1965). Translation of Acad. Rep. Populare Romine, Studii Cercetari Energet., Ser. A, 12, 513-531, 1962.

G. F. Popper, *Counting and Campbelling: A New Approach to Neutron-Detection Systems*, Reactor Fuel Process, 10, No. 3, 199-207 (Summer 1967).

T. A. Porsching, *The Numerical Solution of the Reactor Kinetics Equations by Difference Analogs: A Comparison of Methods*, WAPD-TM-564 (Mar. 1966).

T. A. Porsching, *Numerical Solution of the Reactor Kinetics Equations by Approximate Exponentials*, Nucl. Sci. Eng., 25, No. 2, 183-188 (June 1966).

J. Prades and Y. Panis, *Digital Computers in Nuclear Power Applications*, IRE Trans. Nucl. Sci., NS-8, No. 4, 119-126 (Oct. 1961).

SELECTED NUCLEAR BIBLIOGRAPHY

F. Reisch, *Stability Analysis of a Sampled-Data Controlled Nuclear Reactor System*, Nucl. Sci. Eng., 26, No. 3, 378-384 (Nov. 1966).

R. P. Remshaw and L. E. Weaver, *State Variable Feedback Design of a Control System for a Coupled-Core Reactor*, NASA-CR-95406 (June 1968).

R. B. Rice et al., *Automatic Control of T7 Tanker Boiling Water Reactor Propulsion System*, GEAP-3528 (Rev. 1) (Sept. 1960).

C. W. Ricker, *Digital Computer Control of Power Reactors*, Nucl. Safety, 5, No. 1, 62-63 (Fall 1963).

C. W. Ricker, *Measurement of Reactor Fluctuation Spectra and Subcritical Reactivity*, Ph.D. Thesis, Univ. Michigan, 1965. (UM No. 66-5108)

J. J. Roberts and H. P. Smith, *On-line Computers in Optimal Reactor Shutdown*, IEEE Trans. Nucl. Sci., NS-13, No. 1, 448-453 (Feb. 1966).

J. J. Roberts, *Time Optimal Shutdown of a Nuclear Reactor*, Ph.D. Thesis, Univ. California, 1966. (UM No. 66-8371)

J. C. Robinson, *Analysis of Neutron Fluctuation Spectra in the Oak Ridge Research Reactor and the High Flux Isotope Reactor*, ORNL-4149 (Oct. 1967).

G. Roselli, *Application of Direct Digital Control*, RT/ING(66)16 (1966). (In Italian)

T. Rosescu, *Stochastic Processes in Nuclear Reactor Theory-Bibliography*, IFA-FR-47 (Dec. 1965).

Z. R. Rosztoczy, *Optimal Xenon Shutdown Control of Nuclear Reactors*, Trans. Amer. Nucl. Soc., 10, No. 1, 256-257 (June 1967).

K. Saito, *Noise-Equivalent Source in Nuclear Reactors*, Nucl. Sci. Eng., 28, No. 3, 384-396 (June 1967).

K. Saito, *On the Noise-Equivalent Source in a Zero-Power Reactor*, Nucl. Sci. Eng., 28, No. 3, 452-456 (June 1967).

K. Saito and Y. Taji, *Theory of Branching Processes of Neutrons in a Multiplying Medium*, Nucl. Sci. Eng., 30, No. 1, 54-64 (Oct. 1967).

T. D. Schmidt, *A Digital Start-up Control Unit for Nuclear Reactors*, IRE Trans. Nucl. Sci., NS-8, No. 3, 1-12 (July 1961).

H. Schow, *A Detailed Derivation of the Courant-Wallace Expressions for the Second Moments of the Neutron Probability Distribution and for the Variance/Mean Ratio*, AFRRI TN65-1 (Oct. 1965).

SELECTED NUCLEAR BIBLIOGRAPHY

H. Schow, *The Numerical Evaluation of the Theoretical Variance/Mean Ratio of the Neutron Population in the Subcritical Condition of the AFRRI-TRIGA Reactor*, AFRRI TN66-1 (Jan. 1966).

J. G. Selmeczy and A. Camplani, *On-line Computer Generates Enrico-Fermi Performance Data*, Elec. World, 164, No. 6, 64-66 (Aug. 1965).

J. R. Sheff, *A General Equation for the Cross-Correlation of the Counts in Two Separate Time Intervals*, Trans. Amer. Nucl. Soc., 7, No. 2, 278 (Nov. 1964).

Y. Shinohara and J. Valat, *Optimization of Xenon Poisoning by Optimization of Build-up or Over-ride*, ORNL-TR-419 (Mar. 1965).

Y. Shinohara, K. Monta, and K. Sato, *Optimal Control Problems in Nuclear Reactor Engineering; III: Applications*, J. At. Energy Soc. Jap., 10, No. 2, 89-96 (Feb. 1968). (In Japanese)

E. Siddall, *Computer Control of Nuclear Power Plants*, Proc. Eng. Joint Council Nucl. Cong. (New York, June 1962), Paper No. 29.

E. Siddall and J. E. Smith, *Computer Control in the Douglas Point Nuclear Power Station*, AECL-2948 (Sept. 1967).

H. B. Smets, *Reactor Dynamics at Low Power*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1958), 11, 237-244.

H. B. Smets, *Problems in Nuclear Power Reactor Stability*, (Brussels: University of Brussels Press, 1962).

H. B. Smets, *Stability in the Large and Boundedness of Some Reactor Models*, Reactor Sci. Technol., 17, 329-340 (1963).

H. B. Smets, *Derivation of the Describing Function for a Reactor at Power Level*, Nukleonik, 7, No. 7, 399-405 (Aug. 1965).

H. P. Smith, *Dynamics and Control of Nuclear Rocket Engines*, Ph.D. Thesis, Massachusetts Institute of Technology, 1960.

H. Soodak, *Pile Kinetics*, Science and Engineering of Nuclear Power, 2, (Reading, Mass.: Addison-Wesley Publishing Company, 1949), 89-102.

D. H. Stegemann, *Determination of Physical Reactor Parameters from Reactor Noises, by Probability-Distribution Analysis*, EURFNR-285 (Jan. 1967).

B. G. Strait and G. L. Hohmann, *Automatic Startup for Nuclear Reactor Rocket Engines*, Proc. IFAC Symp. (Stavanger, Norway, June 1965), (New York: Plenum Press, 1966), 415-421.

B. G. Strait and R. M. Lang, *Nuclear Reactor Control with Stepping Motor Actuators*, LADC-7244 (May 1966).

SELECTED NUCLEAR BIBLIOGRAPHY

N. Suda, *Optimal Control Problems in Nuclear Reactor Engineering; II: Optimization Techniques*, J. At. Energy Soc. Jap., 10, No. 2, 87-89 (Feb. 1968). (In Japanese)

N. Suda, *Optimal Control Problems in Nuclear Reactor Engineering; I: Incentives and Problem Formulation*, J. At. Energy Soc. Jap., 10, No. 2, 85-86 (Feb. 1968). (In Japanese)

A. Sumner, *Optimal Identification of Some Parameters of a Nuclear Reactor Dynamical System*, Proc. IFAC, 1, Bk. 2, Paper No. 26A (1966).

M. Surdin and C. Kassimatis, *Correlation-Function Analysis of Systems Obeying Linear Differential Equations*, Noise Analysis in Nuclear Systems, TID-7679 (June 1964), 321-329.

P. Szulc and J. Podgorski, *Recent Development in Control and Safety Systems for Nuclear Reactors*, Nukleonika, 9, No. 7-8, 551-562 (1964).

T. Taguchi and K. Takumi, *An Automatic Startup of HTR with a Digital System*, J. Nucl. Sci. Technol., 4, No. 7, 321-327 (July 1967).

Y. Takahashi and S. Takamatsu, *Digital Start-up Control of a Research Reactor*, IEEE Trans. Nucl. Sci., NS-12, No. 4, 355-366 (Aug. 1965).

Y. Takahashi, S. Takamatsu, and K. Monta, *Computer Control of Research Reactor TTR-1*, Toshiba Review (Tokyo), No. 25, 37-43 (Spring 1966).

S. Takamatsu and K. Monta, *Computer Control of Research Reactor TTR-1*, Trans. Amer. Nucl. Soc., 10, No. 1, 368-369 (June 1967).

S. Takamatsu, *The Computer Control of Nuclear Reactors*, J. Soc. Instrum. Contr. Eng., 7, No. 3, 172-179, (March 1968). (In Japanese)

K. Takumi and T. Taguchi, *Digital Computer Control of a Research Reactor*, IEEE Trans. Nucl. Sci., NS-15, No. 1, 87-92 (Feb. 1968).

S. Tan, *Solution of Reactor Kinetics Equations by B.W.K. Approximation*, Nukleonik, 8, No. 8, 480-483 (1966).

W. B. Terney and H. Fenech, *Control-Rod Programming Optimization Using Dynamic Programming*, Trans. Amer. Nucl. Soc., 11, No. 1, 354-355 (June 1968).

J. A. Thie, *Statistical Analysis of Power-Reactor Noise*, Nucleonics, 17, No. 10, 102-111 (Oct. 1959).

J. A. Thie, *Reactor Noise*, (New York: Rowman and Littlefield, Inc., 1963).

J. A. Thie, *Elementary Methods of Reactor Noise Analysis*, Nucl. Sci. Eng., 15, No. 2, 109-114 (Feb. 1963).

SELECTED NUCLEAR BIBLIOGRAPHY

J. A. Thie, *Noise Sources in Power Reactors*, Noise Analysis in Nuclear Systems, TID-7679 (June 1964), 357-368.

J. A. Thie, *Stochastic Processes in Nuclear Reactors and Measurement of Dynamic Performance Characteristics*, Proc. Int. Conf. Peaceful Uses At. Energy (Geneva, Switzerland, 1964), 4, 54-60.

I. Thierer, *The Application of Volterra Series and Nonlinear Operators to Nuclear Reactor Kinetics*, Ph.D. Thesis, Univ. Florida, 1967. (UM No. 68-9559)

R. K. Thomasson, *The Control of Large Fast Reactors*, IEE Symp. Automat. Contr. Elect. Supply (Manchester, England, March 1966), 344-349.

T. J. Thompson and J. G. Beckerley (eds.), *The Technology of Nuclear Reactor Safety; 1: Reactor Physics and Control*, (Cambridge, Mass.: M.I.T. Press, 1964).

J. R. Trinko, Jr. and S. H. Hanauer, *Ion-Chamber Current Fluctuations Produced by Neutron and Gamma Sources*, Trans. Am. Nucl. Soc., 8, No. 1, 60-61 (June 1965).

J. R. Trinko, *An Investigation of the Limitations of a Pulse Type Neutron Detector for the Analysis of Reactor Noise*, Ph.D. Thesis, Univ. Tennessee, 1967. (UM No. 68-9834)

E. Turkcan and J. B. Dragt, *Experimental Study of Different Techniques for Analysing Reactor Noise Measured by a Neutron Counter*, RCN-75 (June 1967).

H. Tuxen-Meyer, *The On-line Digital Computer for the Marviken Nuclear Power Station*, Proc. ISA 9th Nat. Power Instr. Symp. (Detroit, Mich., May 1966), 9, 31-36.

C. Velez, *Autocorrelation Functions of Counting Rate in Nuclear Reactors*, Nucl. Sci. Eng., 6, No. 5, 414-419 (Nov. 1959).

G. C. Vellender, *A Feasibility Study of the Application of a Digital Data Acquisition and Process Control System to a Research Reactor*, ANL-7009 (Jan. 1965).

F. Velona, *Main Functions of Digital Computers in the Enel Nuclear Power Plants*, Energ. Nucl. (Milan), 14, No. 1, 24-31 (Jan. 1967).

C. H. Vincent, J. B. Rowles, and R. A. W. Steels, *A Precise Digital Period Meter for a Nuclear Reactor*, Nucl. Instr. Methods, 26, 221-237 (1964).

C. H. Vincent, *The Digital Reactor Period Meter*, Nucl. Instr. Methods, 31, 345-346 (Dec. 1964).

SELECTED NUCLEAR BIBLIOGRAPHY

H. E. Voress, *Nuclear Rockets: A Bibliography, 1966-1967*, TID-3586 (Aug. 1968).

J. F. Walter, *Detector Response to the Subcritical Reactor*, Nucl. Appl., 3, No. 5, 271-274 (1967).

L. E. Weaver and H. S. Murray, *Mean-Square Error Minimization of Reactor Noise*, TID-7662 (Apr. 1964).

J. Weill and J. Furet, *New Method for the Fast Start of Nuclear Reactors*, Compt. Rend. Acad. Sci. (Paris), 258, Groupe 6, 887-888 (Jan. 1964). (In French)

D. Welbourne, *Data Processing and Control by a Computer at Wylfa Nuclear Power Station*, Proc. Inst. Mech. Engrs., 179, Part 3H, 92-104 (1965).

N. S. Wells, *Fault Experience on the NRU Dual-Computer System*, Trans. Amer. Nucl. Soc., 9, No. 1, 264-265 (June 1966).

D. M. Wiberg, *Optimal Feedback Control of Spatial Xenon Oscillations in a Nuclear Reactor*, TID-21273 (June 1964).

D. M. Wiberg, *Optimal Control of Nuclear Reactors*, Advances in Control Systems, 5, (ed.) C. T. Leondes (New York: Academic Press, 1967), 301-388.

G. R. Woodcock and A. L. Babb, *Optimal Reactor Shutdown Programs for Control of Xenon Poisoning*, Trans. Amer. Nucl. Soc., 8, No. 1, 235 (June 1965).

J. G. Yevick and A. Amorosi (eds.),*Fast Reactor Technology: Plant Design*, (Cambridge, Mass.: M.I.T. Press, 1966).

B. A. Zolotar, *Monte Carlo and Experimental Analyses of Nuclear Reactor Fluctuation Models*, Ph.D. Thesis, Cornell Univ., 1967. (UM No. 68-683)

V. G. Zolotukhin and A. I. Mogilner, *Distribution of the Counting of a Neutron Detector Placed in a Reactor*, At. Energ. (USSR), 10, No. 4, 379-381 (Apr. 1961).

V. G. Zolotukhin and A. I. Mogilner, *The Distribution of Counts from a Detector Placed in a Reactor*, At. Energ. (USSR), 15, No. 1, 664-670 (May 1964).

SELECTED CONTROL BIBLIOGRAPHY

N. Abbattista et al., *A PDP-8 Computer Control System*, Nucl. Instrum. Methods, 62, 337-340 (July 1968).

L. R. Abramson and K. S. Miller, *Minimum Variance Estimation and Prediction Theory*, Int. J. Eng. Sci., 5, No. 3, 251-263 (Mar. 1967).

A. E. Albert and L. A. Gardner, *Stochastic Approximation and Nonlinear Regression*, (Cambridge, Mass.: M.I.T. Press, 1967).

P. Alper, *A Consideration of the Discrete Volterra Series*, IEEE Trans. Automat. Contr., AC-10, No. 3, 322-327 (July 1965).

B. G. Anderson, *The Optimization of Computer-Controlled Systems Using Partial Knowledge of the Output State*, Proc. IFAC, 1, 480-486 (1963).

A. Ia. Andrienko, *The Method of Statistical Optimization of Nonlinear Discrete-Time Automatic Control Systems*, Eng. Cybern., 120-130 (July-Aug. 1967).

M. Aoki, *Optimization of Stochastic Systems, Topics in Discrete-Time Systems*, (New York: Academic Press, 1967).

M. Aoki, *Optimal Bayesian and Min-Max Control of a Class of Stochastic and Adaptive Dynamic Systems*, Proc. IFAC Symp. (Tokyo, 1965), 77-85.

M. Aoki, *Optimal Control of Some Class of Imperfectly Known Control Systems*, J. Basic Eng., 88, No. 2, 306-310 (June 1966).

M. Aoki, *On Observability, Identifiability and Controllability of Stochastic Discrete-Time Dynamic Systems*, Proc. NEC, 22, 821-826 (1966).

M. Aoki and J. R. Huddle, *Estimation of the State Vector of a Linear Stochastic System with a Constrained Estimator*, Proc. JACC, 694-702 (1966).

M. Aoki and M. T. Li, *Optimal Discrete-Time Stochastic Control Systems with Constrained Observation and Control Schemes*, Proc. Hawaii Int. Conf. Syst. Sci. (Honolulu, Jan. 1968), 47-50.

S. Arimoto, *Linear, Stationary, Optimal Feedback Control Systems*, Inform. Contr., 9, No. 1, 79-93 (Feb. 1966).

R. Aris, *Discrete Dynamic Programming*, (Waltham, Mass.: Blaisdell Publishing Co., 1964).

E. A. Aronson, *A Method of Identification of Certain Linear Control Systems*, SC-R-742 (Dec. 1963).

J. A. Aseltine, *Non-linear Sampled-Data System Analysis by the Incremental Phase-Plane Method*, Proc. IFAC, 1, 295-304 (1960).

SELECTED CONTROL BIBLIOGRAPHY

M. Ash, *Application of Dynamic Programming to Stochastic Time Optimal Control*, IEEE Int. Conv. Rec., Pt. I, 212-215 (1964).

K. J. Astrom, R. W. Koepcke and F. Tung, *On the Control of Linear Discrete Dynamic Systems with Quadratic Loss*, RJ-222 (Sept. 1962).

K. J. Astrom, *Optimal Control of Markov Processes with Incomplete State Information*, J. Math. Anal. Appl., 10, No. 1, 174-205 (Feb. 1965).

K. J. Astrom, *Computer Control of a Paper Machine - An Application of Linear Stochastic Control Theory*, IBM J. Res. Develop., 11, No. 4, 389-405 (July 1967).

M. Athans, P. L. Falb, and R. T. Lacoss, *Optimal Control of Self-Adjoint Systems*, IEEE Trans. Appl. Ind. 83, No. 71, 161-166 (May 1964).

M. Athans, *The Status of Optimal Control Theory and Applications for Deterministic Systems*, IEEE Trans. Automat. Contr. AC-11, No. 3, 580-596 (July 1966).

M. Athans and P. L. Falb, *Optimal Control: An Introduction to the Theory and Its Application*, (New York: McGraw-Hill Book Co., 1966).

M. Athans, *On the Equivalence of Linearized Kalman Filters*, ESD-TR-67-20 (Jan. 1967).

M. Athans, *The Matrix Minimum Principle*, NASA-CR-89628 (Aug. 1967).

M. Athans, R. P. Wishner, and A. Bertolini, *Suboptimal State Estimation for Continuous-Time Nonlinear Systems from Discrete Noisy Measurements*, Proc. JACC, 364-382 (1968).

J. Auricoste and R. Vichnevetsky, *The Use of On-line Digital Computers in Process Control*, Progr. Contr. Eng., 3, 197-239 (1966).

A. Avez, *Ergodic Theory of Dynamical Systems*, Univ. Minnesota, School Math. (1966).

F. N. Bailey, *Vector Lyapunov Functions for a Class of Interconnected Systems*, Proc. NEC, 21, 593-598 (1965).

A. V. Balakrishnan, *A General Theory of Non-linear Estimation Problems in Control Systems*, J. Math Anal. Appl., 8, No. 1, 4-30 (Feb. 1964).

A. V. Balakrishnan and L. W. Neustadt (eds.) *Computing Methods in Optimization Problems*, (New York: Academic Press, 1964).

A. V. Balakrishnan and L. W. Neustadt (eds.) *Mathematical Theory of Control*, (New York: Academic Press, 1967).

A. V. Balakrishnan, *Communication Theory*, (New York: McGraw-Hill Book Co., 1968).

SELECTED CONTROL BIBLIOGRAPHY

E. Balas, *Duality in Discrete Programming*, Stanford Univ., Dept. Operations Res. Tech. Rep. No. 67-5 (July 1967).

S. G. Bankoff, *Some Generalizations of the Discrete Optimal Control Problem*, Northwestern Univ. (Oct. 1966).

H. T. Banks, *A Maximum Principle for Optimal Control Problems with Functional Differential Systems*, NASA-CR-95958 (Aug. 1968).

J. G. P. Barnes, *An Algorithm for Solving Nonlinear Equations Based on the Secant Method*, Comput. J., 8, 66-72 (April 1965).

S. Barnett and C. Storey, *Insensitivity of Optimal Linear Control Systems to Persistent Changes in Parameters*, Int. J. Contr., 4, No. 2, 179-184 (1966).

M. S. Bartlett, *An Introduction to Stochastic Processes*, (New York: Cambridge University Press, 1962).

M. Becker, *The Principles and Applications of Variational Methods*, (Cambridge, Mass.: M.I.T. Press, 1964).

H. M. Beisner, *Recursive Bayesian Method for Estimating States of Nonlinear System from Sequential Indirect Observations*, IEEE Trans. Syst. Cybern. SSC-3, No. 2, 101-105 (Nov. 1967).

R. Bellman, *On the Application of the Theory of Dynamic Programming to the Study of Control Processes*, Proc. Symp. Nonlinear Circuit Analysis, 6, (Polytechnic Inst., Brooklyn, New York, 1956), 199-213.

R. Bellman, *Dynamic Programming and Stochastic Control Processes*, Inform. Contr., 1, No. 3, 228-239 (Sept. 1958).

R. Bellman and R. Kalaba, *On Adaptive Control Processes*, IRE Trans. Automat. Contr., AC-4, No. 2, 1-9 (Nov. 1959).

R. Bellman and R. Kalaba, *Dynamic Programming and Adaptive Processes: Mathematical Foundation*, IRE Trans. Automat. Contr., AC-5, No. 1, 5-10 (Jan. 1960).

R. Bellman and R. Kalaba, *Dynamic Programming and Feedback Control*, Proc. IFAC, 1, 460-464 (1960).

R. Bellman (ed.) *Symposium on Mathematical Optimization Techniques*, (Berkeley: University of California Press, 1963).

R. E. Bellman et al., *Invariant Imbedding and Nonlinear Filter Theory*, RM-4374-PR (Dec. 1964).

SELECTED CONTROL BIBLIOGRAPHY

R. Bellman and R. Kalaba (eds.) *Selected Papers on Mathematical Trends in Control Theory*, (New York: Dover Publications, Inc., 1964).

R. E. Bellman, H. H. Kagiwada, and R. E. Kalaba, *Quasilinearization, Boundary-value Problems and Linear Programming*, IEEE Trans. Automat. Contr., AC-10, No. 2, 199 (Apr. 1965).

R. Bellman and R. Kalaba, *Quasilinearization and Nonlinear Boundary-value Problems*, (New York: American Elsevier Publishing Co., Inc., 1965).

R. Bellman, *Dynamic Programming, System Identification, and Suboptimization*, SIAM J. Contr., 4, No. 1, 1-5 (Feb. 1966).

R. Bellman, *Introduction to the Mathematical Theory of Control Processes, I* (New York: Academic Press, 1967).

J. Benes, *Statistical Dynamics of Automatic Control Systems*, (London: ILIFFE Books Ltd., 1967).

A. Ben-Israel, *A Newton-Raphson Method for the Solution of Systems of Equations*, J. Math. Anal., Appl., 15, No. 2, 243-252 (Aug. 1966).

W. R. Bennett, *Methods of Solving Noise Problems*, Proc. IRE, 44, No. 5, 609-638 (May 1956).

J. Bernadyn, G. Pfersch, and J. McGough, *Off-line Adaptive Flight Control System*, AFFDL-TR-65-182 (Apr. 1966).

J. Bernamont, *Fluctuations of Potential at the Terminals of a Metallic Conductor of Small Volume Traversed by a Current*, Ann. Phys. (Paris) 7, 71-140 (1937). (In French).

J. W. Bernard and J. F. Cashen, *Direct Digital Control - Questions that Must be Answered*, Annu. ISA Conf. Proc., Paper No. 5.4-1-64 (1964).

J. W. Bernard and J. F. Cashen, *Direct Digital Control*, Instrum. Contr. Syst. 38, No. 9, 151-158 (Sept. 1965).

J. E. Bertram and P. E. Sarachik, *On Optimal Computer Control*, Proc. IFAC, 1, 419-422 (1960).

J. E. Bertram, *The Direct Method of Lyapunov in the Analysis and Design of Discrete-Time Control Systems*, Work Session in Lyapunov's Second Method, (ed.) L. F. Kazda (Ann Arbor, Mich.: University of Michigan, 1960), 79-104.

J. E. Bertram, *The Concept of State in the Analysis of Discrete-Time Control Systems*, State Space Techniques for Control Systems Workshop, AIEEE (1962).

SELECTED CONTROL BIBLIOGRAPHY

N. P. Bhatia and G. P. Szego, *Dynamical Systems: Stability Theory and Applications*, (Berlin: Springer-Verlag, 1967).

T. A. Bickart, *Matrix Exponential: Approximation by Truncated Power Series*, Proc. Hawaii Int. Conf. Syst. Sci. (Honolulu, Jan. 1968), 523-526.

D. R. Bjork, *Digital Computer Program for the Simulation of Filters and Other Transfer Functions*, SC-TM-67-144 (Apr. 1967).

A. Blaquiere and G. Leitmann, *Further Geometric Aspects of Optimal Processes: Multiple-Stage Dynamic Systems*, Mathematical Theory of Control, (eds.) A. V. Balakrishnan and L. W. Neustadt (New York: Academic Press, 1967), 143-155.

R. Boudarel, J. Delmas, and P. Guichet, *Optimal Control of Processes, I*, (Paris: Dunod, 1967). (In French).

P.I. Boulton and R. J. Kavanagh, *A Method of Producing Multiple Non-correlated Random Signals from a Single Gaussian Noise Source*, Proc. JACC, Paper 14-1 (1962).

J. E. Bradt, *Kalman Filter Study*, D2-82589-1 (July 1966).

S. M. Brainin and G. A. Bekey, *Estimates of the Statistics of Randomly Varying Parameters of Linear Systems*, NASA-CR-79759 (June 1966).

K. B. Brammer, *Low Order Linear Filtering and Prediction of Nonstationary Random Sequences*, SRL-67-0003 (Feb. 1967).

L. Braun and J. G. Truxal, *Adaptive Control*, Appl. Mech. Rev., 17, No. 7, 501-508 (July 1964).

J. V. Breakwell, J. L. Speyer and A. E. Bryson, *Optimization and Control of Nonlinear Systems Using the Second Variation*, SIAM J. Contr., 1, No. 2, 193-223 (1963).

D. L. Briggs and J. R. Schmid, *Lizard, A Program to Solve Non-linear Ordinary Differential Equations*, KAPL-M-EC-36 (Oct. 1963).

K. R. Brown and G. W. Johnson, *Rapid Computation of Optimal Trajectories*, IBM J. Res. Develop., 11, No. 4, 373-382 (July 1967).

A. E. Bryson, *Optimal Programming and Control*, Proc. IBM Sci. Comput. Symp. Contr. Theory Appl. (Yorktown Heights, N. Y., Oct. 1964) 3-24 (1966).

A. E. Bryson and L. J. Henrikson, *Estimation Using Sampled-data Containing Sequentially Correlated Noise*, NASA-CR-87502 (June 1967).

SELECTED CONTROL BIBLIOGRAPHY

R. S. Bucy, *Comment on the Kalman Filter and Nonlinear Estimates of Multivariate Normal Processes*, IEEE Trans. Automat. Contr., AC-10, No. 1, 118 (Jan. 1965).

R. S. Bucy, *Optimal Filtering for Correlated Noise*, RM-5107-PR (Sept. 1966).

T. E. Bullock, *Computation of Optimal Controls by a Method Based on Second Variations*, NASA-CR-84847 (Dec. 1966).

J. R. Burnett and R. K. Whitford, *Digital Computers in the Synthesis of Nonlinear Feedback Systems*, Proc. Symp. Nonlinear Circuit Analysis, 6 (Polytechnic Inst. Brooklyn, New York, 1956), 255-272.

A. M. Bush, *Some Techniques for the Synthesis of Nonlinear Systems*, Massachusetts Inst. Tech. Res. Lab. Electronics (Mar. 1966).

A. G. Butkovskii, *The Necessary and Sufficient Conditions for Optimality of Discrete Control Systems*, Automat. Remote Contr., 24, No. 8, 963-970 (Jan. 1964).

J. A. Cadzow, *Optimization of Sampled-Data Systems*, Ph.D. Thesis, Cornell Univ., 1964. (UM No. 65-3331).

J. A. Cadzow, *Quadratic Optimization of Linear Discrete Systems by Elementary Matrix Theory*, ISA Trans., 6, No. 1, 54-58 (Jan. 1967).

J. A. Cadzow, *Minimum Energy Regulator for a Linear Discrete System*, IEEE Trans. Automat. Contr., AC-12, No. 2, 183-185 (Apr. 1967).

J. A. Cadzow, *An Extension of the Minimum Norm Controller for Discrete Systems*, IEEE Trans. Automat. Contr., AC-12, No. 2 202-203 (Apr. 1967).

E. R. Caianiello (ed.) *Functional Analysis and Optimization*, (New York: Academic Press, 1966).

D. A. Calahan, *A Stable, Accurate Method of Numerical Integration for Nonlinear Systems*, Proc. IEEE, 56, No. 4, 744 (Apr. 1968).

M. D. Canon and J. H. Eaton, *A New Algorithm for a Class of Quadratic Programming Problems with Application to Control*, SIAM J. Contr., 4, No. 1, 34-35 (Feb. 1966).

T. M. Carney and R. M. Goldwyn, *Numerical Experiments with Various Optimal Estimators*, J. Optimiz. Theory Appl., 1, 113-130 (Sept. 1967).

F. P. Caruthers and H. Levenstein, *Adaptive Control Systems*, (New York: Pergammon Press, 1963).

S. S. L. Chang, *Digitized Maximum Principle*, Proc. IRE, 48, No. 12, 2030-2031 (Dec. 1960).

SELECTED CONTROL BIBLIOGRAPHY

S. S. L. Chang, *Dynamic Programming and Pontryagin's Maximum Principle*, AFOSR-865 (June 1961).

S. S. L. Chang, *Synthesis of Optimum Control Systems*, (New York: McGraw-Hill Book Co., 1961).

S. S. L. Chang, *Computer Optimization of Nonlinear Control Systems by Means of Digitized Maximum Principle*, IRE Int. Conv. Rec., 9, Pt. 4, 48-55 (1961).

S. S. L. Chang, *A Modified Maximum Principle for Optimum Control of a System with Bounded Phase Space Coordinates*, Proc. IFAC, 2, 358-362 (1963).

S. S. L. Chang, *On the Relative Time of Adaptive Processes*, Proc. NEC, 20, 606-611 (1964).

S. S. L. Chang, *General Theory of Optimal Processes with Applications*, Proc. IFAC Symp. (Tokyo, 1965), 87-95.

S. S. L. Chang, *General Theory of Optimal Processes*, SIAM J. Contr., 4, No. 1, 46-55 (Feb. 1966).

S. S. L. Chang, *On Convexity and the Maximum Principle for Discrete Systems*, IEEE Trans. Automat. Contr., AC-12, No. 1, 121-122 (Feb. 1967).

S. S. L. Chang, *Discrete Systems and Digital Computer Control*, Appl. Mech. Rev., 20, No. 5, 429-437 (May 1967).

Chi-Tsong Chen, *On the Stability of Non-linear Sampled-Data Feedback Systems*, J. Franklin Inst. 280, No. 4, 316-324 (Oct. 1965).

C. T. Chen, C. A. Desoer, and A. Niederlinski, *Simplified Conditions for Controllability and Observability of Linear Time-Invariant Systems*, IEEE Trans. Automat. Contr., AC-11, No. 3, 613-614 (July 1966).

K. Chen, *Quasi-linearization Design of Nonlinear Feedback Control Systems*, IEEE Trans. Appl. Ind., 83, No. 72, 189-194 (May 1964).

K. Chen and K. H. Bhavnani, *Optimization of Time-Dependent Systems by Dynamic Programming*, ISA Trans., 6, No. 2, 157-161 (Apr. 1967).

Y. Chu, *Digital Analog Simulation Techniques*, NASA-CR-83551 (Feb. 1967).

D. H. Chyung, *Discrete Linear Optimal Control Systems with Essentially Quadratic Cost Functionals*, IEEE Trans. Automat. Contr., AC-11, No. 3 (July 1966).

D. H. Chyung, *An Approximation to Bounded Phase Coordinate Control Problem for Linear Discrete Systems with Quadratic Cost Functionals*, Proc. JACC, 63-69 (1966).

SELECTED CONTROL BIBLIOGRAPHY

D. H. Chyung, *An Approximation to Bounded Phase Coordinate Control Problem for Linear Discrete Systems*, IEEE Trans. Automat. Contr., AC-12, No. 1, 37-42 (Feb. 1967).

D. H. Chyung, *Nonunique Discrete Optimal Control*, Proc. Hawaii Int. Conf. Syst. Sci. (Honolulu, Jan. 1968), 45-46.

N. W. Clark, *A Study of Some Numerical Methods for the Integration of Systems of First-Order Ordinary Differential Equations*, ANL-7428 (Mar. 1968).

J. E. Clough and A. W. Westerberg, *Fortran for On-line Control*, Contr. Eng., 15, No. 3, 77-81 (Mar. 1968).

T. C. Coffey, *The Application of Modern Computing Technology to Control System Analysis Design Problems*, SAMSO-TR-67-9 (June 1967).

M. M. Connors, *Controllability of Discrete, Linear, Random, Dynamical Systems*, SIAM J. Contr., 5, No. 2, 183-210 (May 1967).

R. Conti, *A Bibliography of Optimum Control Theory, Year 1962*, Int. J. Contr., 1, No. 4, 327-333 (Apr. 1965).

G. R. Cooper and J. C. Lindenlaub, *Adaptive Control Review; Vol. II: Limits on the Identification Time for Linear Systems*, ASD-TDR-61-28-V2 (Sept. 1965).

H. L. Cornish and W. L. Horton, *Computerized Process Control*, (New York: Hobbs, Dorman & Co., 1968).

R. L. Cosgriff, *Nonlinear Control Systems*, (New York: McGraw-Hill Book Co., 1958).

D. R. Cox and H. D. Miller, *The Theory of Stochastic Processes*, (London: Methuen & Co. Ltd., 1965).

H. Cox, *Estimation of State Variables for Noisy Dynamic Systems*, Ph.D. Thesis, Massachusetts Institute of Technology, 1963.

H. Cox, *On the Estimation of State Variables and Parameters for Noisy Dynamic Systems*, IEEE Trans. Automat. Contr., AC-9, 5-12 (Jan. 1964).

H. Cox, *Estimation of State Variables via Dynamic Programming*, Proc. JACC, 376-381 (1964).

H. Cramer and M. R. Leadbetter, *Stationary and Related Stochastic Processes*, (New York: John Wiley & Sons, Inc., 1967).

K. E. Cross, *A Gradient Projection Method for Constrained Optimization*, K-1746 (May 1968).

SELECTED CONTROL BIBLIOGRAPHY

I. G. Cumming, *Numerical Techniques for the Non-linear Prediction Problem*, Automatica, 3, Nos. 3/4, 257-273 (Jan. 1966).

H. B. Curry, *The Method of Steepest Descent for Nonlinear Minimization Problems*, Quart. Appl. Math., 2, No. 3, 258-261 (Oct. 1944).

R. E. Curry, *Estimation and Control with Quantized Measurements*, NASA-CR-95764 (May 1968).

E. B. Dahlin, *On-line Identification of Process Dynamics*, IBM J. Res. Develop., 11, No. 4, 406-426 (July 1967).

H. D'Angelo and T. J. Higgins, *A Time-Domain Procedure for the Analysis of Time-Variant Sampled-Data Automatic Control Systems*, ISA Trans., 5, No. 3, 248-255 (July 1966).

W. C. Davidon, *Variable Metric Method for Minimization*, ANL-5990 (1959).

L. D. Davisson, *The Filtering of Time Series with Unknown Signal Statistics*, Proc. NEC, 21, 506-516 (1965).

L. D. Davisson, *The Adaptive Prediction of Time Series*, Proc. NEC, 22, 557-561 (1966).

J. Dawkins, *Control Measurement in the Presence of Noise*, J. Electron. Contr., 15, No. 3, 245-268 (Sept. 1963).

W. DeBacker, *The Generalized Gradient, Its Computational Aspects and Its Relations to the Maximum Principle*, EUR 411.e (1963).

W. DeBacker, *The Maximum Principle, Its Computational Aspects and Its Relations to Other Optimization Techniques*, EUR 590.e (1964).

R. J. P. de Figueiredo and L. W. Dyer, *On Discrete Non-linear Filtering and Dynamic Stochastic Approximation*, Rice Univ. (June 1968).

N. Declaris and N. Liskov, *A Synthesis Method for Linear-Time Varying Systems*, Proc. NEC, 21, 599-604 (1965).

W. J. Dejka, *The Generation of Discrete Functions within a Digital Computer*, IRE Trans. Automat. Contr., AC-7, 56-57 (July 1962).

W. J. Dejka and W. V. Kershaw, *Digital Sampled-Data Feedback Systems for Shipboard Applications*, NEL-1142 (Oct. 1962).

E. V. Denardo, *Contraction Mappings in the Theory Underlying Dynamic Programming*, RM-4755-PR (Mar. 1966).

W. F. Denham and J. L. Speyer, *Optimal Measurement and Velocity Correction Programs for Midcourse Guidance*, AIAA J., 2, No. 5, 896-907 (May 1964).

SELECTED CONTROL BIBLIOGRAPHY

W. F. Denham and S. Pines, *Sequential Estimation When Measurement Function Nonlinearity is Comparable to Measurement Error*, AIAA J., 4, No. 6, 1071-1076 (June 1966).

M. M. Denn, *Discrete Maximum Principle*, Ind. Eng. Chem. Fundamentals, 4, No. 2, 240 (May 1965).

J. E. Dennis, *On Newton's Method and Nonlinear Simultaneous Displacements*, SIAM J. Numer. Anal., 4, No. 1, 103-108 (1967).

P. M. DeRusso, *Optimum Linear Filtering of Signals Prior to Sampling*, AIEE Trans. Appl. Ind., 79, No. 52, 549-555 (Jan. 1961).

C. A. Desoer, *An Introduction to State-Space Techniques in Linear Systems*, State Space Techniques for Control Systems Workshop, AIEE (1962).

C. A. Desoer, E. Polak, and J. Wing, *Theory of Minimum Time Regulators*, Proc. IFAC, 2, 135-141 (1963).

D. M. Detchmendy, *Identification, Estimation, and Control of Nonlinear Systems*, Ph.D. Thesis, Purdue Univ., 1965. (UM No. 66-2252)

D. M. Detchmendy and R. Sridhar, *Sequential Estimation of States and Parameters in Noisy Nonlinear Dynamical Systems*, J. Basic Eng., 88, No. 2, 362-368 (June 1966).

D. M. Detchmendy and R. Sridhar, *On the Experimental Determination of the Dynamical Characteristics of Physical Systems*, Proc. NEC, 21, 575-580 (1965).

R. Deutsch, *Nonlinear Transformation of Random Processes*, (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962).

A. G. Dewey, *Frequency Domain Stability Criteria for Nonlinear Multivariable Systems*, Advances in Computer Control (IEE Conf. Pub. No. 29), Paper No. C12 (Apr. 1967).

J. J. Deyst, *Optimal Control in the Presence of Measurement Uncertainties*, NASA-CR-90504 (Jan. 1967).

J. J. Deyst, *Optimal Control of Linear Stochastic Systems with Measurement Errors*, NASA-CR-94291 (Feb. 1968).

J. J. Deyst, *Conditions for Asymptotic Stability of the Discrete, Minimum Variance, Linear, Estimator*, NASA-CR-94292 (Mar. 1968).

J. L. Doob, *Stochastic Processes*, (New York: John Wiley & Sons, Inc., 1953).

SELECTED CONTROL BIBLIOGRAPHY

R. C. Dorf, *Time-pomain Analysis and Design of Control Systems*, (Reading, Mass.: Addison-Wesley Publishing Co., 1965).

R. C. Dorf, *Modern Control Systems*, (Reading, Mass.: Addison-Wesley Publishing Co., 1967).

P. H. Dosik, *Synthesis of Optimal Control Systems*, Electro-Technology, 75, No. 2, 36-43 (Feb. 1965).

H. J. Dougherty, *Synthesis of Optimal Feedback Control Systems Subject to Modelling Inaccuracies*, Ph.D. Thesis, Rensselaer Polytechnic Institute, 1966. (UM No. 67-4163)

R. F. Drenick and L. Shaw, *Optimal Control of Linear Plants with Random Parameters*, IEEE Trans. Automat. Contr., AC-9, 3, 236-244 (July 1964).

R. M. Dressler, *A Simplified Technique for the Synthesis of Model-Referenced Adaptive Control Systems*, SU-SEL-66-004 (Mar. 1966).

R. M. Dressler, *An Approach to Model-Referenced Adaptive Control Systems*, Proc. NEC, 22, 535-539 (1966).

S. E. Dreyfus, *Dynamic Programming and the Calculus of Variations*, J. Math. Anal. Appl., 1, 228-239 (1960).

S. E. Dreyfus, *Variational Problems with State Variable Inequality Constraints*, P-2605-1 (Aug. 1963).

S. E. Dreyfus, *On the Dynamic Programming Treatment of Discrete-Time Variational Problems*, RM-5159-PR (Dec. 1966).

S. E. Dreyfus, *Some Aspects of the Relationship of Dynamic Programming to the Calculus of Variations*, Proc. IBM Sci. Comput. Symp. Contr. Theory Appl. (Yorktown Heights, N. Y., Oct. 1964) 45-52 (1966).

S. E. Dreyfus, *The Numerical Solution of Non-linear Optimal Control Problems*, Numerical Solution of Nonlinear Equations, (ed.) D. Greenspan (New York: John Wiley & Sons, Inc., 1966), 97-113.

S. E. Dreyfus, *Dynamic Programming and the Hamilton-Jacobi Method of Classical Mechanics*, RM-5116-1-PR (July 1967).

S. E. Dreyfus, *Introduction to Stochastic Optimization and Control*, Stochastic Optimization and Control, (ed.) H. F. Karreman (New York: John Wiley & Sons, Inc., 1968), 3-23.

A. E. Durling, *Computational Aspects of Dynamic Programming in Higher Dimensions*, Ph.D. Thesis, Syracuse Univ., 1964. (UM No. 65-3417)

A. E. Durling, *Some Computational Aspects of Dynamic Programming in High Dimensions*, Proc. IFAC Symp. (Tokyo, 1965), 107-119.

SELECTED CONTROL BIBLIOGRAPHY

R. P. Eddy, *Generalized Covariances and Power Spectra for Discrete Time Series*, NSRDC-2413 (June 1967).

B. R. Eisenberg, *Quasilinearization, Identification, and Specific Optimal Control*, Ph.D. Thesis, Univ. Florida, 1965. (UM No. 66-8829)

B. R. Eisenberg and A. P. Sage, *Closed Loop Optimization of Fixed Configuration Systems*, Int. J. Control, 3, No. 2, 183-194 (Feb. 1966).

O. I. Elgerd, *Control Systems Theory*, (New York: McGraw-Hill Book Co., 1967).

D. H. Eller and J. K. Aggarwal, *Sub-optimal Control of Non-linear Single Input Systems*, Int. J. Contr., First Series, 8, No. 2, 113-121 (Aug. 1968).

F. J. Ellert, *Performance Indices for Linear Systems Based on Standard Forms*, Proc. NEC, 22, 643-648 (1966).

T. W. Ellis and A. P. Sage, *Applications of a Method for On Line Estimation and Control*, Proc. 20th Annual Southwestern IEEE Conf. (Houston, Texas, Apr. 1968), Paper No. 15D.

J. L. Engvall, *An Engineering Expose of Numerical Integration of Ordinary Differential Equations*, NASA-TN-D-3696 (Nov. 1966).

V. W. Eveleigh, *Adaptive Control and Optimization Techniques*, (New York: McGraw-Hill Book Co., 1967).

L. T. Fan et al., *A Sequential Union of the Maximum Principle and Dynamic Programming*, J. Electron. Contr., 17, No. 5, 593-600 (Nov. 1964).

L. T. Fan and C. S. Wang, *The Discrete Maximum Principle; A Study of Multistage Systems Optimization*, (New York: John Wiley & Sons, 1964.

L. T. Fan and C. S. Wang, *The Discrete Maximum Principle*, Ind. Eng. Chem. Fundamentals, 4, No. 2, 239 (May 1965).

A. F. Fath, *Approximation to the Time-Optimal Control of Linear State-Constrained Systems*, D1-82-0675 (Oct. 1967).

A. F. Fath and T. J. Higgins, *Fixed-Time Fuel-Optimal Control of Linear State-Constrained Systems by Use of Linear Programming Techniques*, Proc. JACC, 462-467 (1968).

A. A. Fel'dbaum, *Dual Control Theory 1*, Automat. Remote Contr., 21, No. 9, 874-880 (Sept. 1960).

A. A. Fel'dbaum, *Dual Control Theory 2*, Automat. Remote Contr., 21, No. 11, 1033-1039 (Nov. 1960).

SELECTED CONTROL BIBLIOGRAPHY

A. A. Fel'dbaum, *Dual Control Theory 3*, Automat. Remote Contr., 22, No. 1, 1-12 (Jan. 1961).

A. A. Fel'dbaum, *Dual Control Theory 4*, Automat. Remote Contr., 22, No. 2, 109-121 (Feb. 1961).

A. A. Fel'dbaum, *Optimal Control Systems*, (New York: Academic Press, 1965).

A. A. Fel'dbaum et al., *Theoretical Fundamentals of Communication and Control*, FTD-HT-67-191 (Aug. 1967).

H. A. Fertik and J. D. Schoeffler, *The Design of Nonlinear Multivariable Systems from State-Dependent Linear Models*, J. Basic Eng., 88, No. 2, 355-361 (June 1966).

F. A. Ficken, *Linear Transformations and Matrices*, (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967).

F. A. Fine and S. G. Bankoff, *Second-Variational Methods for Optimization of Discrete Systems*, Ind. Eng. Chem., Fundament., 6, No. 2, 293-299 (May 1967).

D. T. Finkbeiner, *Introduction to Matrices and Linear Transformations*, (San Francisco: W. H. Freeman and Co., 1966).

J. R. Fisher, *Optimal Nonlinear Filtering*, Advances in Control Systems, 5, (ed.) C. T. Leondes (New York: Academic Press, 1967), 301-388.

W. H. Fleming and M. Nisio, *On the Existence of Optimal Stochastic Controls*, J. Math. Mech., 15, 777-794 (1966).

R. Fletcher, *Generalized Inverse Methods for the Best Squares Solution of Systems of Non-linear Equations*, Comput. J., 10, No. 4, 392-399 (Feb. 1968).

J. J. Florentin, *Optimal Control of Systems with Generalized Poisson Inputs*, Proc. JACC, Paper 14-2 (1962).

T. Fortmann, *Optimal Piecewise Constant Solutions of the Linear Regulator Problem*, NASA-CR-92740 (Oct. 1967).

T. R. Fredriksen, *Direct Digital Processor Control of Stepping Motors*, IBM J. Res. Developm., 11, No. 2, 179-188 (Mar. 1967).

E. A. Freeman, *Design of Optimal Linear Control Systems with Quadratic Performance Indices*, Proc. IEE, 114, No. 8, 1180-1183 (Aug. 1967).

H. Freeman, *On the Information-Handling Efficiency of a Digital Computer Program*, AIEE Trans. Appl. Ind., 79, No. 52, 502-508 (Jan. 1961).

SELECTED CONTROL BIBLIOGRAPHY

M. Freimer, *A Dynamic Programming Approach to Adaptive Control Processes*, IRE Trans. Automat. Contr., AC-4, No. 2, 10-15 (Nov. 1959).

B. Friedland, *Optimum Control of Discrete-Time Dynamic Processes*, Proc. IFAC, 2, 128-134 (1963).

B. Friedland, *Optimum Control of Discrete-Time Dynamic Processes*, Proc. JACC, 93 (1963).

B. Friedland, *A Technique of Quasi-optimum Control*, Proc. JACC, 244-252 (1965).

B. Friedland and I. Bernstein, *Estimation of the State of a Nonlinear Process in the Presence of Non-Gaussian Noise and Disturbances*, J. Franklin Inst., 281, No. 6, 455-480 (June 1966).

B. Friedland, F. E. Thau, and P. E. Sarachik, *Stability Problems in Randomly-Excited Dynamic Systems*, Proc. JACC, 848-861 (1966).

B. Friedland and P. E. Sarachik, *A Unified Approach to Suboptimal Control*, Proc. IFAC, 1, Bk. 1, Paper No. 13A (1966).

B. Friedland, *On the Effect of Incorrect Gain in Kalman Filter*, IEEE Trans. Automat. Contr., AC-12, No. 5, 610 (Oct. 1967).

B. Friedland et al., *Additional Studies of Quasi-optimum Feedback Control Techniques*, NASA-CR-1099 (July 1968).

J. C. Friedly, *Asymptotic Approximations to Plug Flow Process Dynamics*, Proc. JACC, 216-228 (1967).

T. Fukao, *System Identification by Bayesian Learning Process*, Proc. IFAC Symp. (Tokyo, 1965), 137-146.

A. T. Fuller, *Bibliography of Optimum Non-linear Control of Determinate and Stochastic-Definite Systems*, J. Electron. Contr., 13, No. 6, 589-611 (Dec. 1962).

A. T. Fuller, *The Replacement of Saturation Constraints by Energy Constraints in Control Optimization Theory*, Int. J. Contr., First Series, 6, No. 3, 201-227 (Sept. 1967).

R. E. Funderlic, *The Programmer's Handbook: A Compendium of Numerical Analysis Utility Programs*, K-1729 (Feb. 1968).

H. Furstenberg, *Stationary Processes and Prediction Theory*, (Princeton, N. J.: Princeton University Press, 1961).

R. Gabasov and F. M. Kirillova, *Extending L. S. Pontryagin's Maximum Principle to Discrete Systems*, Automat. Remote Contr., 27, No. 11, 1878-1882 (Nov. 1966).

SELECTED CONTROL BIBLIOGRAPHY

W. L. Garrard, N. H. McClamroch, and L. G. Clark, *An Approach to Sub-optimal Feedback Control of Non-linear Systems*, Int. J. Contr., First Series, 5, 425-435 (May 1967).

M. K. Gavurin, *Nonlinear Functional Equations and Continuous Analogs of Iterative Methods*, Univ. Maryland, Computer Science Center, Tech. Rep. No. 68-70 (June 1968).

C. W. Gear, *The Numerical Integration of Ordinary Differential Equations of Various Orders*, ANL-7126 (Jan. 1966).

J. E. Gibson, *Nonlinear Automatic Control*, (New York: McGraw-Hill Book Co., 1963).

J. E. Gibson, *Adaptive Learning Systems*, Proc. NEC, 18, 795-799 (1962).

C. Giese, *State Variable Difference Methods for Digital Simulation*, Simulation, 8, No. 5, 263-269 (May 1967).

M. Girault, *Stochastic Processes*, (New York: Springer-Verlag, 1966).

J. N. Gittelman, *Optimal Control of Discrete-Time, Random Parameter Systems*, Ph.D. Thesis, Univ. Michigan, 1967. (UM No. 68-7609)

K. W. Goff, *Dynamics in Direct Digital Control; I: Estimating Characteristics and Effects of Noisy Signals*, ISA J., 13, No.11, 45-49 (Nov. 1966).

K. W. Goff, *Dynamics in Direct Digital Control; II: A Systematic Approach to DDC Design*, ISA J., 13, No. 12, 44-54 (Dec. 1966).

M. J. Goldstein, *Use of the Implicit Function Theorem in Solving Systems of Equations*, USL-858 (Feb. 1968).

M. J. Goldstein, *Solving Systems of Linear Equations by Using the Generalized Inverse*, USL-874 (Mar. 1968).

E. M. Grabbe, *Digital Computer Control Systems - An Annotated Bibliography*, Proc. IFAC, 2, 1074-1087 (1960).

B. B. Gragg, Jr., *The Computation of Approximately Optimal Control*, Ph.D. Thesis, Stanford Univ., 1964. (UM No. 64-9824)

D. Graham and D. McRuer, *Analysis of Nonlinear Control Systems*, (New York: John Wiley & Sons, Inc., 1961).

D. Graupe and G. R. Cassir, *Adaptive Control by Predictive Identification and Optimization*, Proc. NEC, 22, 590-594 (1966).

SELECTED CONTROL BIBLIOGRAPHY

D. Graupe and G. R. Cassir, *Control of Multivariable Systems by Predictive Optimization*, Advances in Computer Control (IEE Conf. Pub. No. 29), Paper No. C3 (Apr. 1967).

D. Graupe, B. H. Swanick, and G. R. Cassir, *Application of Regression Analysis to Reduction of Multivariable Control Problems and to Process Identification*, Proc. NEC, 23, 20-25 (1967).

J. H. Gray and L. B. Rall, *NEWTON: A General Purpose Program for Solving Nonlinear Systems*, MRC-790 (Sept. 1967).

L. P. Grayson, *The Design of Nonlinear and Adaptive Systems via Lyapunov's Second Method*, PIMBRI-937-61 (1961).

C. J. Greaves and J. A. Cadzow, *The Optimal Discrete Filter Corresponding to a Given Analog Filter*, IEEE Trans. Automat. Contr., AC-12, No. 3, 304-307 (June 1967).

A. L. Greensite, *Dynamic Programming and the Kalman Filter Theory*, Proc. NEC, 20, 601-605 (1964).

A. L. Greensite, *Analysis and Design of Space Vehicle Flight Control Systems, Vol. XIII: Adaptive Control*, NASA-CR-832 (Aug. 1967).

U. Grenander, *Adaptive Stochastic Control*, Differential Equations and Dynamic Systems, (eds.) J. K. Hale and J. P. LaSalle (New York: Academic Press, 1967), 83-102.

J. J. G. Guignabodet, *Some Bounds on Quantization Errors in Dynamic Programming Computations*, Proc. IFAC, 2, 383-385 (1963).

I. Gumowski and C. Mira, *Optimization in Control Theory and Practice*, (London: Cambridge University Press, 1968).

T. L. Gunckel, *Optimum Design of Sampled-Data Systems with Random Parameters*, Stanford Univ., Tech. Rep. No. 2102-2 (Apr. 1961).

T. L. Gunckel and G. F. Franklin, *A General Solution for Linear Sampled-Data Control*, J. Basic Eng., 85, No. 2, 197-203 (June 1963).

S. C. Gupta and C. W. Ross, *Simulation Evaluation of a Digital Control System*, ISA Trans., 3, No. 3, 271-279 (July 1964).

A. H. Haddad and J. B. Thomas, *On Optimal and Suboptimal Nonlinear Filters for Discrete Inputs*, IEEE Trans. Inform. Theory, IT-14, No. 1, 16-21 (Jan. 1968).

G. Hadley, *Linear Algebra*, (Reading, Mass.: Addison-Wesley Publishing Co., 1961).

G. Hadley, *Nonlinear and Dynamic Programming*, (Reading, Mass.: Addison-Wesley Publishing Co., 1964).

SELECTED CONTROL BIBLIOGRAPHY

W. Hahn, *Theory and Application of Liapunov's Direct Method*, (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963).

W. Hahn, *Stability of Motion*, (Berlin: Springer-Verlag, 1967).

R. L. Haken and A. W. Naylor, *A New Method of Designing Time-Varying, Linear Feedback Systems*, Int. J. Cont., 4, No. 2, 137-151 (1966).

J. K. Hale and J. P. LaSalle, *Differential Equations and Dynamical Systems*, (New York: Academic Press, 1967).

H. Halkin, *On the Necessary Condition for Optimal Control of Nonlinear Systems*, Ph.D. Thesis, Stanford, Univ., 1963. (Um No. 64-1611)

H. Halkin, *Optimal Control Systems Described by Difference Equations*, Advances in Control Systems, 1, (ed.) C. T. Leondes (New York: Academic Press, 1964), 173-196.

H. Halkin, *A Maximum Principle of the Pontryagin Type for Systems Described by Nonlinear Difference Equations*, SIAM J. Contr., 4, No. 1, 90-111 (Feb. 1966).

H. Halkin, *General Necessary Conditions for Optimization Problems*, AFOSR-67-0145 (Oct. 1966).

H. Halkin. et al., *Theory of Optimum Discrete-Time Systems*, Proc. IFAC, Paper No. 28B (1968).

K. Hansen, *Bibliography on the Application of Digital Computers in Power Stations 1962-1967*, RISO-M-725 (Mar. 1968).

B. Hanus, *Simplified Reproduction of the Controller Action of a Steam Superheater on the Analog Computer*, ORNL-TR-1802 (1965).

C. A. Harvey, *Synthesis of Time-Optimal Control for Linear Process*, J. Math. Anal. Appl., 10, No. 2, 334-341 (Apr. 1965).

R. A. Harvey, *Analog Technique for Optimizing Dynamic Systems*, BNWL-392 (May 1967).

S. Hayashi and T. Okada, *Optimum Sampled-Data Control System Design by Dynamic Programming Technique*, Mem. Fac. Eng., Kyoto Univ., 28, Pt. 1, 13-32 (Jan. 1966).

R. M. Hayes, *An Algorithm for the Determination of the Definiteness of a Real Square Matrix*, NASA-TM-X-53644 (Aug. 1967).

H. Hermes et al., *Research in the General Area of Non-linear Dynamical Systems*, NASA-CR-61629 (June 1967).

SELECTED CONTROL BIBLIOGRAPHY

G. P. Herring, *The Solution of Generalized Multilinear Regression and Least Squares Function Approximation Problems*, HSM-RO-7-66 (Sept. 1966).

M. R. Hestenes, *Calculus of Variations and Optimal Control Theory*, (New York: John Wiley & Sons, Inc., 1966).

T. J. Higgins, *Stability Analysis of Nuclear Reactors by Lyapunov's Second Method*, Work Session in Lyapunov's Second Method (ed.) L. F. Kazda (Ann Arbor, Mich.: University of Michigan Press, 1960), 107-121.

C. G. Hilborn and D. G. Lainiotis, *Linear Estimation Procedures for Non-Markovian Random Processes*, Proc. NEC, 21, 662-665 (1965).

D. M. Himmelblau and K. B. Bischoff, *Process Analysis and Simulation; Deterministic Systems*, (New York: John Wiley & Sons, Inc., 1968).

Y. C. Ho, *On Stochastic Approximation and Optimum Filtering Methods*, J. Math. Anal. Appl., 6, No. 1, 152-154 (Feb. 1963).

Y. C. Ho and P. B. Brentani, *On Computing Optimal Control with Inequality Constraints*, SIAM J. Contr., 1, No. 3, 319-348 (1963).

Y. C. Ho and R. C. K. Lee, *A Bayesian Approach to Problems in Stochastic Estimation and Control*, IEEE Trans. Automat. Contr., AC-9, No. 4, 333-339 (Oct. 1964).

Y. C. Ho and R. C. K. Lee, *Identification of Linear Dynamic Systems*, Proc. NEC, 20, 647-651 (1964).

L. L. Hoberock and R. H. Kohr, *An Experimental Determination of Differential Equations to Describe Simple Nonlinear Systems*, Proc. JACC, 616-623 (1966).

L. L. Hoberock, *An Experimental Method for the Determination of Nonlinear Differential Equations to Describe Simple Dynamic Systems*, Ph.D. Thesis, Purdue Univ., 1966. (UM No. 66-13207)

J. K. Holmes, *System Identification from Noise-Corrupted Measurements*, J. Optimiz. Theory Appl., 2, No. 2, 102-116 (Mar. 1968).

J. M. Holtzman, *Convexity and the Maximum Principle for Discrete Systems*, IEEE Trans. Automat. Contr., AC-11, No. 1, 30-35 (Jan. 1966).

J. M. Holtzman, *On the Maximum Principle for Nonlinear Discrete-Time Systems*, IEEE Trans. Automat. Contr., AC-11, No. 2, 273-274 (Apr. 1966).

S. G. Hoppe, H. P. Semmelhack, and C. W. Swonger, *A Feasibility Study of Self-learning Adaptive Flight Control for High Performance Aircraft*, AFFDL-TR-67-18 (Feb. 1967).

SELECTED CONTROL BIBLIOGRAPHY

F. Horn and R. Jackson, *Discrete Maximum Principle*, Ind. Eng. Chem. Fundamentals, 4, No. 1, 110-112 (Feb. 1965).

R. E. Horton, *Use of Continuous Measurements in a Discrete Kalman Filter*, Ph.D. Thesis, Iowa State Univ., 1967. (UM No. 67-8914)

K. J. B. Hosking, *Dynamic Programming and Synthesis of Linear Optimal Control Systems*, Proc. IEE, 113, No. 6, 1087-1090 (June 1966).

J. A. Hrastar, *A Computer Control System for an Advanced OAO*, NASA-TM-X-55895 (Aug. 1967).

H. C. Hsieh, *The Least Squares Estimation of Linear and Nonlinear System Weighting Function Matrices*, Inform. Contr., 7, No. 1, 84-115 (Mar. 1964).

J. C. Hsu and A. U. Meyer, *Modern Control Principles and Applications*, (New York: McGraw-Hill Book Co., 1968).

J. C. Hung and A. M. Revington, *Minimum Energy Design of Discrete-Data Control Systems*, Proc. IFAC Symp. (Tokyo, 1965), 23-29.

E. S. Ibrahim and Z. V. Rekasius, *A Stability Criterion for Nonlinear Feedback Systems*, IEEE Trans. Automat. Contr., AC-9, No. 2, 154-159 (Apr. 1964).

J. D. Irwin and J. C. Hung, *Kalman Estimator and Sample Mean*, IEEE Trans. Automat. Contr., AC-12, No. 4, 472 (Aug. 1967).

J. D. Irwin and J. C. Hung, *Estimation of Nonlinear Dynamics*, Proc. NEC, 23, 137-140 (1967).

D. Isaacs and C. T. Leondes, *Optimal Control System Synthesis for Cost Functionals Involving Convex Single Valued Functions of the State and Control Variables*, Inform. Contr., 9, No. 4, 393-413 (Aug. 1966).

D. Isaacs, C. T. Leondes, and R. A. Niemann, *On a Sequential Optimization Approach in Nonlinear Control*, Proc. JACC, 158-166 (1966).

D. Isaacs, *Algorithms for Sequential Optimization of Control Systems*, Advances in Control Systems, 4, (ed.) C. T. Leondes (New York: Academic Press, 1966), 1-71.

K. Ito and M. Nisio, *On Stationary Solutions of a Stochastic Differential Equation*, J. Math. (Kyoto), 4, 1-75 (1964).

R. P. Iwens and A. R. Bergen, *On Bounded-Input Bounded-Output Stability of a Certain Class of Nonlinear Sampled-Data Systems*, J. Franklin Inst., 282, No. 4, 193-205 (Oct. 1966).

SELECTED CONTROL BIBLIOGRAPHY

R. Jackson and F. Horn, *On Discrete Analogues of Pontryagin's Maximum Principle*, Intern. J. Control, 1, No. 4, 389-396 (Apr. 1965).

R. Jackson and F. Horn, *Discrete Maximum Principle*, Ind. Eng. Chem. Fundamentals, 4, No. 4, 487-488 (Nov. 1965).

O. L. R. Jacobs, *An Introduction to Dynamic Programming*, (London: Chapman and Hall Ltd., 1967).

D. H. Jacobson, *New Second-Order and First-Order Algorithms for Determining Optimal Control: A Differential Dynamic Programming Approach*, NASA-CR-93690 (Feb. 1968).

V. K. Jain, *Optimization of Linear Systems*, Ph.D. Thesis, Michigan State Univ., 1964. (UM No. 65-1756)

V. K. Jain and H. E. Koenig, *On Parameter Optimization of Linear Systems*, IEEE Trans. Automat. Contr., AC-11, No. 4, 753-754 (Oct. 1966).

A. H. Jazwinski, *Optimal Trajectories and Linear Control of Nonlinear Systems*, AIAA J., 2, No. 8, 1371-1379 (Aug. 1964).

A. H. Jazwinski, *Adaptive Filtering*, NASA-CR-84794 (Mar. 1967).

A. H. Jazwinski, *Filtering for Nonlinear Dynamical Systems*, IEEE Trans. Automat. Contr., AC-11, No. 4, 765-766 (Oct. 1966).

K. W. Jenkins and R. J. Roy, *A Design Procedure for Discrete Adaptive Control Systems*, Proc. JACC, 624-633 (1966).

S. Jian and H. King-Ching, *Analysis and Synthesis of Time-Optimal Control Systems*, Proc. IFAC, 2, 347-351 (1963).

S. Jian and H. King-Ching, *Analysis and Synthesis of Time Optimal Control for Linear Systems with Variable Coefficients*, Sci. Sinica (Peking), 13, No. 6, 993-1004 (1964).

C. D. Johnson, *Optimal Control with Chebyshev Minimax Performance Index*, J. Basic Eng., 89, No. 2, 251-262 (June 1967).

B. W. Jordan and E. Polak, *Theory of Class of Discrete Optimal Control Systems*, J. Electron. Contr., 17, No. 6, 697-711 (Dec. 1964).

B. Jordan and E. Polak, *Optimal Control of Aperiodic Discrete-Time Systems*, SIAM J. Contr., 2, No. 3, 332-346 (Jan. 1965).

P. Joseph, J. Lewis, and J. Tou, *Plant Identification in the Presence of Disturbances and Application to Digital Adaptive Systems*, AIEE Trans. Appl. Ind., 80, No. 53, 18-24 (Mar. 1961).

P. D. Joseph and J. T. Tou, *On Linear Control Theory*, AIEE Trans. Appl. Ind., 80, No. 56, 193-196 (Sept. 1961).

SELECTED CONTROL BIBLIOGRAPHY

E. I. Jury, *Sampled-Data Control Systems*, (New York: John Wiley & Sons, Inc., 1958).

E. I. Jury, *Recent Advances in the Field of Sampled-Data and Digital Control Systems*, Proc. IFAC, 1, 262-269 (1960).

E. I. Jury, *A Simplified Stability Criterion for Linear Discrete Systems*, Proc. IRE, 50, No. 6, 1493-1500 (June 1962).

E. I. Jury, *On the Roots of a Real Polynomial Inside the Unit Circle and a Stability Criterion of Linear Discrete Systems*, Proc. IFAC, 2, 142-153 (1963).

E. I. Jury and B. W. Lee, *On the Stability of a Certain Class of Nonlinear Sampled-Data Systems*, IEEE Trans. Automat. Contr., AC-9, No. 1, 51-61 (Jan. 1964).

E. I. Jury and T. Nishimura, *Stability of PWM Feedback Systems*, J. Basic Eng., 86, No. 1, 80-86 (Mar. 1964).

E. I. Jury, *Comments on the Statistical Design of Linear Sampled-Data Feedback Systems*, IEEE Trans. Automat. Contr., AC-10, No. 2, 215-217 (Apr. 1965).

E. I. Jury, *On the Stability Condition of Nonlinear Sampled-Data Systems*, IEEE Trans. Automat. Contr., AC-10, No. 2, 217-218 (Apr. 1965).

H. H. Kagiwada et al., *Invariant Imbedding and Sequential Interpolating Filters for Nonlinear Processes*, RM-5507-PR (Nov. 1967).

R. E. Kalman, *Nonlinear Aspects of Sampled-Data Systems*, Proc. Symp. Nonlinear Circuit Analysis, 6, (Polytechnic Inst. Brooklyn, New York, 1956), 273-313.

R. E. Kalman and R. W. Koepcke, *Optimal Synthesis of Linear Sampling Control Systems Using Generalized Performance Indexes*, ASME Trans., 80, No. 8, 1820-1826 (Nov. 1958).

R. E. Kalman and J. E. Bertram, *General Synthesis Procedure for Computer Control of Single-Loop and Multiloop Linear Systems (An Optimal Sampling System)*, AIEE Trans. Appl. Ind., 77, Pt. II, 602-609 (Jan. 1959).

R. E. Kalman and J. E. Bertram, *A Unified Approach to the Theory of Sampling Systems*, J. Franklin Inst., 267, No. 5, 405-436 (May 1959).

R. E. Kalman, *Contributions to the Theory of Optimal Control*, Bol. Soc. Mat. Mexicana, 2nd Ser., 5, No. 1, 102-119 (Apr. 1960).

R. E. Kalman and J. E. Bertram, *Control System Analysis and Design via the Second Method of Lyapunov; II:Discrete-Time Systems*, J. Basic Eng., 82, No. 2, 394-399 (June 1960).

SELECTED CONTROL BIBLIOGRAPHY

R. E. Kalman, T. S. Englar, and R. S. Bucy, *Fundamental Study of Adaptive Control Systems*, ASD-TR-61-27 (Apr. 1962).

R. E. Kalman, Y. C. Ho, and K. S. Narendra, *Controllability of Linear Dynamical Systems*, Contributions to Differential Equations, 1, No. 2, (New York: John Wiley & Sons, Inc., 1963), 189-213.

R. E. Kalman, *When is a Linear Control System Optimal?* J. Basic Eng., 86, No. 1, 51-60 (Mar. 1964).

R. E. Kalman, *Toward a Theory of Difficulty of Computation in Optimal Control*, Proc. IBM Sci. Comput. Symp. Contr. Theory Appl. (Yorktown Heights, N. Y., Oct. 1964), 25-43 (1966).

R. E. Kalman, *Algebraic Aspects of the Theory of Dynamical Systems*, Differential Equations and Dynamic Systems, (eds.) J. K. Hale and J. P. LaSalle (New York: Academic Press, 1967), 133-146.

G. Kang, *Researches in Optimal and Suboptimal Control Theory*, NASA-CR-93737 (Mar. 1968).

K. R. Kaplan and J. Sklansky, *Analysis of Markov Chain Models of Adaptive Processes*, AMRLTR-65-3 (Jan. 1965).

H. F. Karreman (ed.), *Stochastic Optimization and Control*, (New York: John Wiley & Sons, Inc., 1968).

R. L. Kashyap, *Optimization of Stochastic Finite State Systems*, Harvard Univ., Div. Eng. Appl. Physics, Tech. Rep. No. 499 (Apr. 1966).

S. Katz, *A Discrete Version of Pontryagin's Maximum Principle*, J. Electron. Contr., 13, No. 2, 179-184 (Aug. 1962).

J. Katzenelson and L. A. Gould, *The Design of Nonlinear Filters and Control Systems, I*, Inform. Contr., 5, No. 2, 108-143 (June 1962).

J. Katzenelson and L. A. Gould, *The Design of Nonlinear Filters and Control Systems, II*, Inform. Contr., 5, No. 2, 117-145 (June 1964).

A. Kaufmann, *Graphs, Dynamic Programming and Finite Games*, (New York: Academic Press, 1967).

I. E. Kazakov, *Determination of the Distribution Laws for the Variables of a Nonlinear Stochastic System*, Automat. Remote Contr., 26, No. 11, 1859-1869 (Nov. 1965).

L. F. Kazda, *Application of State Variables to Optimal Control System Problems*, State Space Techniques for Control Systems Workshop, AIEE (1962).

SELECTED CONTROL BIBLIOGRAPHY

H. J. Kelley, *Guidance Theory and Extremal Fields*, IRE Trans. Automat. Contr., AC-7, 75-82 (Oct. 1962).

P. Kenneth and G. E. Taylor, *Solution of Variational Problems with Bounded Control Variables by Means of the Generalized Newton-Raphson Method*, Recent Advances in Optimization Techniques, (eds.) A. Lavi and T. P. Vogl (New York: John Wiley & Sons, Inc., 1966), 471-487.

A. S. Kholevo, *Iteration Method for Optimal Nonlinear Filter for a Stationary Random Process*, Eng. Cybern., No. 2, 170-176 (Mar.-Apr. 1967).

R. B. Kieburtz, *The Step Motor - The Next Advance in Control Systems*, IEEE Trans. Automat. Contr., AC-9, No. 1, 98-104 (Jan. 1964).

M. Kim, *On the Minimum Time Control of Linear Sampled-Data Systems*, Proc. IEEE, 53, No. 9, 1263-1264 (Sept. 1965).

M. Kim, *On Optimum Control of Discrete Systems: I. Theoretical Development*, ISA Trans., 5, No. 1, 93-98 (Jan. 1966).

M. Kim, *On Optimum Control of Discrete Systems: II. Hybrid Computer Implementation*, ISA Trans., 5, No. 2, 184-194 (Apr. 1966).

M. Kim and G. U. Ramos, *Implementation of Optimum Sampled-Data Control*, IEEE Trans. Automat. Control., AC-11, No.2, 274-277 (Apr. 1966).

M. Kim and K. Djadjuri, *Optimum Sampled-Data Control Systems with Quantized Control Function*, ISA Trans., 6, No. 1, 65-73 (Jan. 1967).

E. Kinnen and C. Chen, *Lyapunov Functions and the Exact Differential Equation*, NASA-CR-80204 (Dec. 1966).

E. Kinnen and C. S. Chen, *Lyapunov Functions for a Class of Nth Order Nonlinear Differential Equations*, NASA-CR-687 (Jan. 1967).

W. Kipiniak, *Dynamic Optimization and Control, A Variational Approach*, (New York: John Wiley & Sons, Inc., 1961).

D. E. Kirk, *Optimization of Systems with Pulse-Width Modulated Control*, IEEE Trans. Automat. Contr., AC-12, No. 3, 307-309 (June 1967).

K. Kirvaitis and K. S. Fu, *Identification of Nonlinear Systems by Stochastic Approximation*, Proc. JACC, 255-264 (1966).

G. B. Kleindorfer and P. R. Kleindorfer, *Quadratic Performance Criteria with Linear Terms in Discrete-Time Control*, IEEE Trans. Automat. Contr., AC-12, No. 3, 320-321 (June 1967).

D. L. Kleinman and M. Athans, *The Discrete Minimum Principle with Application to the Linear Regulator Problem*, NASA-CR-74852 (Feb. 1966).

SELECTED CONTROL BIBLIOGRAPHY

A. Klinger, *Prior Information and Bias in Sequential Estimation*, P-3655 (Aug. 1967).

F. D. Knight, *Generalized Linear Regression Analysis*, DP-1128 (Feb. 1968).

J. B. Knowles, *An Extreme Value Problem in a Linear Sampled-Data Feedback System*, IEEE Trans. Automat. Contr., AC-8, No. 3, 235-239 (July 1963).

J. B. Knowles, *The Stability of a Proportional Rate Extremum Regulator*, IEEE Trans. Automat. Contr., AC-9, No. 3, 256-264 (July 1964).

J. B. Knowles and R. Edwards, *Aspects of Subrate Digital Control Systems*, Proc. Inst. Elec. Eng., 113, No. 11, 1893-1901 (Nov. 1966).

H. K. Knudsen, *An Iterative Procedure for Computing Time-Optimal Controls*, IEEE Trans. Automat. Contr., AC-9, No. 1, 23-30 (Jan. 1964).

S. Kodama, *Stability of Nonlinear Sampled-Data Control Systems*, IRE Trans. Automat. Contr., AC-7, No. 1, 15-23 (Jan. 1962).

S. Kodama, *Stability of Nonlinear Discrete Control Systems*, Ph.D. Thesis, Univ. California, 1963. (UM No. 64-2082)

H. E. Koenig, Y. Tokad, and H. K. Kesavan, *Analysis of Discrete Physical Systems*, (New York: McGraw-Hill Book Co., 1967).

R. W. Koepcke, *A Solution to the Sampled, Minimum-Time Problem*, J. Basic Eng. 86, No. 1, 145-150 (Mar. 1964).

R. W. Koepcke, *A Discrete Design Method for Digital Control*, Contr. Eng., 13, No. 6, 83-89 (June 1966).

A. J. Koivuniemi, *A Computational Technique for the Design of a Specific Optimal Controller*, IEEE Trans. Automat. Contr., AC-12, No. 2, 180-183 (Apr. 1967).

B. Kondo and S. Iwai, *Analytical Approaches to Non-linear Sampled-Data Control Systems*, Proc. IFAC, 2, 154-161 (1963).

B. Kondo and T. Suzuki, *Optimalizing and Learning Control by a Statistical Model*, Proc. IFAC Symp. (Tokyo, 1965), 69-76.

R. E. Kopp and R. J. Orford, *Linear Regression to System Identification for Adaptive Systems*, J. AIAA, 1, No. 10, 2300-2306 (Oct. 1963).

A. Korsak, *Perturbed Optimal Control Problems*, Univ. California, Berkeley, Dept. Math., AD-486916 (June 1966).

SELECTED CONTROL BIBLIOGRAPHY

K. D. Kotnour, G. E. P. Box, and R. J. Altpeter, *A Discrete Predictor-Controller Applied to Sinusoidal Perturbation Adaptive Optimization*, ISA Trans., 5, No. 3, 255-262 (July 1966).

E. Kounias, *An Adaptive Control Problem in Discrete Time*, Bull. Hellenic Math. Soc., New Series, 7, 61-80 (1966).

E. Kounias, *Optimal Bounded Control with Linear Stochastic Equations and Quadratic Cost*, J. Math. Anal. Appl., 16, No. 3, 510-537 (Dec. 1966).

P. Kovanic, *Optimum Digital Operators*, UJV 1954 (1968).

J. D. R. Kramer, *On Control of Linear Systems with Time Lags*, Inform. Contr., 3, No. 4, 299-326 (Dec. 1960).

N. N. Krasovskii, *Stability of Motion*, (Stanford, Calif.: Stanford University Press, 1963).

N. N. Krasovskii, *Optimal Control Under Discrete Feedback Signals*, Differential Equations, 1, No. 11, 1111-1121, (Nov. 1965).

E. Kriendler, *Reciprocal Optimal Control Problems*, J. Math. Anal. Appl., 14, No. 1, 141-152 (Apr. 1966).

V. F. Krotov, *Sufficient Conditions for the Optimality of Discrete Control Systems*, Soviet Math., 8, No. 1, 11-15 (Jan.-Feb. 1967).

W. H. Kroy, *Identification of Linear Stochastic Dynamical Systems by Nonlinear Weighting of Noisy Measurements*, NASA-CR-84509 (Apr. 1967).

P. D. Krut'ko, *Analytical Design of Digital Regulators by the Method of Dynamic Programming*, Izv. Akad. Nauk SSSR, Tekh. Kibernetika, No. 6, 140-145, (1965). (In Russian)

P. D. Krut'ko, *Analytic Design of Optimal Digital Regulators*, Eng. Cybern., No. 4, 192-198 (Aug. 1966).

P. D. Krut'ko, *Relationship of Dynamic Programming to Lyapunov Functions in Analytic Design of Discrete Systems*, Eng. Cybern., No. 1, 106-111 (Jan.-Feb. 1967).

Y. H. Ku and C. F. Chen, *Stability Study of a Third-Order Servomechanism with Multiplicative Feedback Control*, AIEE Trans. Appl. Ind., 77, No. 3, 131-136 (July 1958).

Y. H. Ku, *Analysis and Control of Linear Systems*, (Scranton, Pa.: International Textbook Co., 1962).

K. S. P. Kumar and R. Sridhar, *A Note on Combined Identification and Control*, IEEE Trans. Automat. Contr., AC-9, No. 11, 118 (Jan. 1964).

SELECTED CONTROL BIBLIOGRAPHY

K. S. P. Kumar and R. Sridhar, *On the Identification of Control Systems by the Quasi-linearization Method,* IEEE Trans. Automat. Contr., AC-9, No. 2, 151-154 (Apr. 1964).

K. S. P. Kumar and R. Sridhar, *On the Identification of Linear Systems,* Proc. JACC, 361-365 (1964).

K. S. P. Kumar, *Identification of Nonlinear, Nonstationary Processes,* Proc. IFAC Symp. (Tokyo, 1965), 237-243.

K. S. P. Kumar et al., *Modern Aspects of Automatic Control, Parts I and II,* (Evanston: Northwestern Univ., 1966).

K. S. P. Kumar, *Discrete Differential Approximation and System Identification,* Int. J. Contr., First Series, 6, 65-73 (July 1967).

H. W. Kuhn and A. W. Tucker, *Nonlinear Programming,* Proc. 2nd Berkeley Symp. on Math. Stat. and Prob. (Univ. California, 1951), 481-492.

H. P. Kunzi, H. G. Tzschach, and C. A. Zehnder, *Numerical Methods of Mathematical Optimization,* (New York: Academic Press, 1968).

B. C. Kuo, *Analysis and Design of Sampled-Data Systems Via State Transition Flow Graphs,* Proc. NEC, 18, 28-39 (1962).

B. C. Kuo, *Design of Relay Type Sampled-Data Control Systems Using Discrete Describing Function,* IRE Int. Conv. Rec., 10, Pt. 2, 99-111 (1962).

B. C. Kuo, *Analysis and Synthesis of Sampled Data Control Systems,* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963).

B. C. Kuo, *Forced Oscillations in Relay-Type Sampled-Data Control Systems,* Proc. NEC, 21, 581-586 (1965).

B. C. Kuo, *Forced Oscillations and Suppression of Oscillations in Nonlinear Sampled Data Systems,* IEEE Trans. Automat. Contr., AC-11, No. 2, 290-292 (Apr. 1966).

F. F. Kuo and J. F. Kaiser (eds.), *System Analysis by Digital Computer,* (New York: John Wiley & Sons, Inc., 1966).

H. J. Kushner, *On the Optimum Timing of Observations for Linear Control Systems with Unknown Initial State,* IEEE Trans. Automat. Contr., AC-9, No. 2, 144-150 (Apr. 1964).

H. J. Kushner, *On the Dynamical Equations of Conditional Probability Density Functions, with Applications to Optimal Stochastic Control Theory,* J. Math. Anal. Appl., 8, 332-344 (1964).

SELECTED CONTROL BIBLIOGRAPHY

H. J. Kushner, *On the Stability of Processes Defined by Stochastic Difference-Differential Equations*, Brown Univ., Div. Applied Math., Tech. Rep. No. 67-4 (Apr. 1967).

H. J. Kushner, *Dynamical Equations for Optimal Nonlinear Filtering*, J. Diff. Equations, 3, No. 2, 179-190 (Apr. 1967).

H. J. Kushner, *Stochastic Stability and Control*, (New York: Academic Press, 1967).

H. J. Kushner, *Approximations to Optimal Non-linear Filters*, NASA-CR-85852 (1967).

P. I. Kuznetsov, R. L. Stratonovich, and V. I. Tikhonov, *Non-linear Transformations of Stochastic Processes*, (New York: Pergammon Press, 1965).

H. Kwakernaak, *On-Line Iterative Optimization of Stochastic Control Systems*, Automatica, 2, No. 3, 195-208 (Jan. 1965).

H. Kwakernaak, *Optimal Filtering in Linear Systems with Time Delays*, IEEE Trans. Automat. Contr., AC-12, No. 2, 169-173 (Apr. 1967).

SELECTED CONTROL BIBLIOGRAPHY

J. H. Laning and R. H. Battin, *Random Processes in Automatic Control*, (New York: McGraw-Hill Book Co., 1956).

L. Lapidus and R. Luus, *The Control of Nonlinear Systems: Part 1, Direct Search on the Performance Index*, A.I.Ch.E.J., 13, No. 1, 101-108 (Jan. 1967).

R. E. Larson, *Dynamic Programming with Reduced Computational Requirements*, IEEE Trans. Automat. Contr., AC-10, No. 2, 135-143 (Apr. 1965).

R. E. Larson and J. Peschon, *A Dynamic Programming Approach to Trajectory Estimation*, IEEE Trans. Automat. Contr., AC-11, No. 3, 537-540 (July 1966).

R. E. Larson, *Optimum Quantization in Dynamic Systems*, IEEE Trans. Automat. Contr., AC-12, No. 2, 162-168 (Apr. 1967).

R. E. Larson, *A Survey of Dynamic Programming Computational Procedures*, IEEE Trans. Automat. Contr., AC-12, No. 6, 767-774 (Dec. 1967).

R. E. Larson, R. M. Dressler, and R. S. Ratner, *Precomputation of the Weighting Matrix in an Extended Kalman Filter*, Proc. JACC, 634-645 (1967).

J. LaSalle and S. Lefschetz, *Stability by Liapunov's Direct Method with Applications* (New York: Academic Press, 1961).

J. P. LaSalle and S. Wan, *Optimality and Strong Stability of Control Systems*, NASA-CR-95574 (June 1968).

G. J. Lastman, *Optimization of Nonlinear Systems with Inequality Constraints*, NASA-CR-82652 (Dec. 1966).

G. J. Lastman, *Newton's Method and the Optimization of Nonlinear Systems*, NASA-CR-87552 (Jan. 1967).

A. Lavi and T. P. Vogl (eds.), *Recent Advances in Optimization Techniques* (New York: John Wiley & Sons, Inc., 1966).

D. F. Lawden, *Optimal Trajectories for Space Navigation* (London: Butterworths, 1963).

D. M. Layton, *Suboptimal Discrete Time Kalman Filtering*, Proc. 5th Allerton Conf. Circuit System Theory (Monticello, Ill., Oct. 1967), 781-790.

E. B. Lee, *A Sufficient Condition in the Theory of Optimal Control*, Siam J. Contr., 1, No. 3, 241-245 (1963).

E. B. Lee, *A Computational Scheme for Discrete Systems*, IEEE Trans. Automat. Contr., AC-9, No. 1, 115 (Jan. 1964).

SELECTED CONTROL BIBLIOGRAPHY

E. B. Lee and L. Markus, *Foundations of Optimum Control Theory* (New York: John Wiley & Sons, Inc., 1967).

E. S. Lee, *Reduction in Dimensionality, Dynamic Programming and Quasilinearization. I. General Theory,* Kansas State Univ. Bull., 51, No. 8 (Aug. 1967).

E. S. Lee et al., *Introduction to Application of Quasilinearization to the Solutions of Non-Linear Differential Equations,* AFOSR-67-1867 (Mar. 1967).

E. S. Lee, *Quasilinearization in Optimization: A Numerical Study,* A.I.Ch.E.J., 13, No. 6, 1043-1051 (Nov. 1967).

E. S. Lee, *Quasilinearization and Invariant Imbedding* (New York: Academic Press, 1968).

E. S. Lee, *Iterative Techniques in Optimization, I. Dynamic Programming and Quasilinearization,* Proc. JACC, 923-939 (1968).

T. H. Lee, G. E. Adams, and W. M. Gaines, *Computer Process Control-Modeling and Optimization* (New York: John Wiley & Sons, Inc., 1968).

Y. W. Lee, *Some Aspects of the Wiener Theory of Nonlinear Systems,* Proc. NEC, 21, 759-764 (1965).

S. Lefschetz, *Stability of Nonlinear Control Systems* (New York: Academic Press, 1965).

S. H. Lehnigk, *Stability Theorems for Linear Motions with an Introduction to Liapunov's Direct Method* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1966).

K. N. Leibovic, *Contraction Mapping with Application to Control Processes,* J. Electron. Contr., 15, No. 1, 81-95 (July 1963).

K. N. Leibovic, *The Principle of Contraction Mapping in Nonlinear and Adaptive Control Systems,* IEEE Trans. Automat. Contr., AC-9, No. 4, 393-398 (Oct. 1964).

G. Leitmann (ed.), *Optimization Techniques with Applications to Aerospace Systems* (New York: Academic Press, 1962).

G. Leitmann (ed.), *Topics in Optimization* (New York: Academic Press, 1966).

G. Leitmann, *Introduction to Optimal Control* (New York: McGraw-Hill Book Co., 1966).

G. G. Lendaris, *On the Identification and Adaptive Control of Sampled-Data Systems,* AFOSR-720 (Mar. 1961).

SELECTED CONTROL BIBLIOGRAPHY

O. A. Z. Leneman and J. B. Lewis, *Random Sampling of Random Processes: Mean-Square Comparison of Various Interpolators*, IEEE, Trans. Automat. Contr., AC-11, No. 3, 396-403 (July 1966).

C. T. Leondes (ed.), *Modern Control Systems Theory* (New York: McGraw-Hill Book Co., 1965).

C. T. Leondes and G. Paine, *Extensions in Quasilinearization Techniques for Optimal Control*, J. Optimiz. Theory Appl., 2, No. 5, 316-330 (Sept. 1968).

Yu. P. Leonov, *Asymptotically Optimal Systems as Models of Adaptive Control*, Soviet Physics - Doklady, 11, No. 3, 200-201 (Sept. 1966).

A. M. Letov, *Stability in Non-Linear Control Systems* (Princeton, N. J.: Princeton University Press, 1961).

A. M. Letov, *The Synthesis of Optimal Regulators*, Proc. IFAC, 2, 246-262 (1963).

V. S. Levadi and R. L. Cosgriff, *A Describing Function for Nonlinear Systems with Memory Subject to Random Input*, IEEE Trans. Appl. Ind., 83, No. 71, 73-76 (Mar. 1964).

K. Levenberg, *A Method for the Solution of Certain Nonlinear Problems in Least Squares*, Quart. Appl. Math., 2, No. 2, 164-168 (July 1944).

L. J. Levy and G. L. Blank, *Characterization of Nonlinear Systems by the Functional Power Series*, Proc. NEC, 23, 164-169 (1967).

J. Lewins, *Optimal System Operation with State Restrictions*, Nucl. Energy, 53-57 (Mar.-Apr. 1967).

T. O. Lewis and P. L. Odell, *A Theory of Linear Estimation*, NASA-CR-62053 (Aug. 1967).

A. M. Liapunov, *Stability of Motion* (New York: Academic Press, 1966).

A. I. Liff, *System Identification in the Presence of Noise by Digital Techniques*, IEEE Int. Conv. Rec., 14, Part 6, 152-166 (Mar. 1966).

R. N. Linebarger, *Digital Simulation Techniques for Direct Digital Control Studies*, Annu. ISA Conf. Proc., Paper No. 5.4-3-64 (1964).

R. Linebarger and R. D. Brennan, *Digital Simulation for Control System Design*, Instrum. Contr. Syst., 38, No. 10, 147-152 (Oct. 1965).

A. L. Ljungwe, *Identification, Prediction, and Optimization for Discrete, Adaptive Control*, Ph.D. Thesis, State Univ. Iowa, 1963. (UM No. 64-3390).

SELECTED CONTROL BIBLIOGRAPHY

P. V. Lopresti, *On the Control of Discrete Random Parameter Systems*, Ph.D. Thesis, Purdue Univ., 1963. (UM No. 64-5744).

B. W. Lovell, *A Digital Computer Program for Control System Analysis*, NASA-CR-91504 (Aug. 1967).

J. M. Lucke, *The Method of Steepest Descent*, PB-176355 (Aug. 1967).

D. L. Lukes, *Stabilizability and Optimal Control*, MRC 822 (Oct. 1967).

R. Luus and L. Lapidus, *The Control of Nonlinear Systems: Part 2, Convergence by Combined First and Second Variations*, A.I.Ch.E.J., 13, No. 1, 108-113 (Jan. 1967).

D. MacKinnon, *Optimal Control of Systems with Pure Time Delays Using A Variational Programming Approach*, IEEE Trans. Automat. Contr., AC-12, No. 3, 255-262 (June 1967).

D. T. Magill, *Optimal Adaptive Estimation of Sampled Stochastic Processes*, Ph.D. Thesis, Stanford Univ., 1964. (UM No. 64-9835).

D. T. Magill, *Optimal Adaptive Estimation of Sampled Stochastic Processes*, IEEE Trans. Automat. Contr., AC-10, No. 4, 434-439 (Oct. 1965).

D. T. Makers, H. B. Haake, and P. Briggs, *Investigation of Advanced Flight Control Systems Through Application of Linear Sensitivity Techniques*, AFFDL-TR-66-142, Vol. I (Oct. 1966).

D. T. Makers and J. J. Carnall, *Digital Mechanization of a Discretely Adaptive Flight Control Computer*, AFFDL-TR-68-34 (Apr. 1968).

M. Mangad and M. D. Schwartz, *Guidance, Flight Mechanics and Trajectory Optimization. Vol. 4: The Calculus of Variations and Modern Applications*, NASA-CR-1003 (Jan. 1968).

O. L. Mangasarian and S. Fromovitz, *A Maximum Principle in Mathematical Programming*, Mathematical Theory of Control (eds.) A. V. Balakrishnan and L. W. Neustadt (New York: Academic Press, 1967), 85-95.

J. Mann, S. A. Marshall, and H. Nicholson, *A Review of Matrix Techniques in Multivariable System Control*, Proc. Inst. Mech. Eng., 179, Pt. 3H, 153-161 (1964-65).

J. P. Mantey, *A Computer-Oriented Study of Performance Feedback in Sampled-Data Control Systems*, Ph.D. Thesis, Stanford Univ., 1965. (UM No. 65-6321).

R. M. Marchetti, *An Approach to Control Optimization for Nonlinear Systems*, J. Spacecraft Rockets, 4, 1096-1098 (Aug. 1967).

SELECTED CONTROL BIBLIOGRAPHY

D. W. Marquardt, *An Algorithm for Least-Squares Estimation of Nonlinear Parameters*, J. Soc. Ind. Appl. Math., 11, No. 2, 431-441 (June 1963).

J. J. Martin, *Bayesian Decision Problems and Markov Chains* (New York: John Wiley & Sons, Inc., 1967).

L. V. Maurin and J. A. Calvo, *Computing Control at Little Gypsy Generating Station*, ISA J., 13, No. 5, 34-38 (May 1966).

D. Q. Mayne, *Optimal Non-Stationary Estimation of the Parameters of a Linear System with Gaussian Inputs*, J. Electron. Contr., 14, No. 1, 101-112 (Jan. 1963).

D. Q. Mayne, *Parameter Estimation*, Automatica, 3, Nos. 3/4, 245-255 (Jan. 1966).

D. Mayne, *A Second-Order Gradient Method for Determining Optimal Trajectories of Non-Linear Discrete-Time Systems*, Int. J. Contr., 3, No. 1, 85-95 (Jan. 1966).

D. Q. Mayne, *A Gradient Method for Determining Optimal Control of Non-Linear Stochastic Systems*, Theory of Self-Adaptive Control Systems (ed.) P. H. Hammond (New York: Plenum Press, 1966), 19-27.

D. Q. Mayne, *A Method for Estimating Discrete Time Transfer Functions*, Advances in Computer Control (IEE Conf. Pub. No. 29), Paper No. C2 (Apr. 1967).

K. A. McCollom and R. M. Stewart, *Adaptive Sampling in Digital Control*, IS-995 (Aug. 1964).

R. McGill and P. Kenneth, *Solution of Variational Problems by Means of a Generalized Newton-Raphson Operator*, NASA-CR-69699 (May 1964).

R. McGill, *Optimal Control, Inequality State Constraints, and the Generalized Newton-Raphson Algorithm*, AD-484711 (July 1965).

J. P. McGuire, *Digital Computer Estimation of System Parameters Utilizing Input-Output Data*, EDL-M-875 (Aug. 1966).

R. W. McLaren, *A Markov Model for Learning Systems Operating in an Unknown Random Environment*, Proc. NEC., 20, 585-589 (1964).

J. D. McLean, S. F. Schmidt, and L. A. McGee, *Optimal Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission*, NASA TN D-1208 (Mar. 1962).

J. C. McMichael and G. C. Skezas, *Case Studies of Optimum Filter-Controller Design in Sampled Data Systems*, AD-483584 (May 1966).

SELECTED CONTROL BIBLIOGRAPHY

C. McMillan and E. Huber, *Time Responses and Parameter Estimation of Nonlinear Control Systems*, EDL-M954 (Jan. 1967).

J. S. Meditch, *Discrete Optimal Filtering*, AD 484379 (Mar. 1965).

J. S. Meditch, *A Class of Suboptimal Linear Controls*, IEEE Trans. Automat. Contr., AC-11, No. 3, 433-439 (July 1966).

J. S. Meditch, *Orthogonal Projection and Discrete Optimal Linear Smoothing*, SIAM J. Contr., 5, No. 1, 74-89 (Feb. 1967).

J. S. Meditch, *Near-Optimal Stochastic Linear Controls*, D1-82-0647 (Sept. 1967).

J. S. Meditch, *Minimum Mean-Square Stochastic Linear Control*, DI-820693 (Apr. 1968).

L. Meier and B. D. O. Anderson, *Performance of Suboptimal Linear Control Systems*, Proc. Inst. Elec. Eng., 114, No. 1, 124-138 (Jan. 1967).

A. Z. Meiri, *On Optimal Filtering and Control of Stochastic Systems*, Proc. IFAC Symp. (Tokyo, 1965), 61-68.

T. Meksawan and G. J. Murphy, *Optimum Design of Nonlinear Sampled-Data Control Systems*, Regelungstechnik, 11, No. 7, 295-299 (1963).

J. L. Melsa, *A Digital Computer Program for the Analysis and Design of State Variable Feedback Systems*, NASA-CR-850 (Aug. 1967).

J. L. Melsa and D. G. Schultz, *A Closed-Loop, Approximately Time-Optimal Control Method for Linear Systems*, IEEE Trans. Automat. Contr., AC-12, No. 1, 94-97 (Feb. 1967).

C. W. Merriam, *An Optimization Theory for Feedback Control System Design*, Inform. Contr., 3, No. 1, 32-59 (Mar. 1960).

C. W. Merriam, *Optimization Theory and the Design of Feedback Control Systems* (New York: McGraw-Hill Book Co., 1964).

W. E. Meserve and D. Jordan, *A Comparison of Time-Optimal Dual and Single Control of a Second-Order Control System*, ISA Trans., 6, No. 2, 147-156 (Apr. 1967).

B. L. Miller, *Finding Optimal Policies in Discrete Dynamic Programming*, RM 5601 PR (Apr. 1968).

W. E. Miller, *Digital Computer Applications to Process Control* (New York: Plenum Press, 1965).

SELECTED CONTROL BIBLIOGRAPHY

R. C. Minnich, *Review of Kalman Filter and Other Algorithms Considered*, Montana State Univ., Eng. Res. Lab. (Sept. 1967).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 1: Continuously Acting Control Systems*, AD 250 **109** (Mar. 1960).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 2: Discontinuous Control Systems*, AD 263 278(Mar. 1961).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 3: Special Problems of Control Theory*, AD 277 189 (Mar. 1962).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 4: Method of Harmonic Linearization*, AD 412 481 (Mar. 1963).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 5: Piece-Wise Linear Methods and Absolute Stability*, AD 628 954 (Mar. 1964).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 6: External Systems and Introduction of Statistical Data*, AD 628 955 (Mar. 1965).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 7: Statistical Trends in Control Theory*, AD 641 261 (Mar. 1966).

N. Minorsky, *Investigation of Nonlinear Control Systems; Part 8: Oscillations in Control Systems*, AD 669 523 (Apr. 1967).

E. Mishkin and L. Braun, *Adaptive Control Systems* (New York: McGraw-Hill Book Co., 1961).

S. K. Mitter, *Successive Approximation Methods for The Solution of Optimal Control Problems*, Automatica, 3, Nos. 3/4, 135-149 (Jan. 1966).

T. Miura, J. Tsuda and J. Iwata, *Hybrid Computer Solution of Optimal Control Problems by the Maximum Principle*, IEEE Trans. Electron. Comput., EC-16, 666-670 (Oct. 1967).

R. V. Monopoli and C. E. Hutchinson, *Estimation of States in Systems with Unknown Parameter Variations*, Proc. IEEE Region 6 Annu. Conf. (Tucson, Ariz., April, 1966).

A. J. Monroe, *Digital Processes in Sampled-Data Control Systems* (New York: John Wiley & Sons, Inc., 1961).

J. B. Moore and B. D. O. Anderson, *Optimal Linear Control Systems with Input Derivative Constraints*, Proc. Inst. Elec. Eng., 114, No. 12, 1987-1990 (Dec. 1967).

A. I. Moroz, *Synthesis Problem of Time-Optimal Control for Discrete Plants*, Automat. Remote Contr., 27, No. 11, 1883-1893 (Nov. 1966).

SELECTED CONTROL BIBLIOGRAPHY

R. E. Mortensen, *Stochastic Optimal Control with Noisy Observations*, Int. J. Contr., 4, No. 5, 455-464 (1966).

R. E. Mortensen, *A Priori Open Loop Optimal Control of Continuous Time Stochastic Systems*, Int. J. Contr., 3, No. 2, 113-127 (Mar. 1966).

R. E. Mortensen, *Optimal Control of Continuous-Time Stochastic Systems*, ERL-66-1 (Aug. 1966).

F. J. Mullin and J. deBabeyrac, *Linear Digital Control*, J. Basic Eng., 86, No. 1, 61-66 (Mar. 1964).

P. A. Muraviev, *A Generalized LaPlace-Carson Transformation and Its Application to the Solution of Linear Differential Equations with Variable Coefficients*, AECL-2817 (Nov. 1966).

G. J. Murphy and T. Meksawan, *Optimal Synthesis of Linear Pulse-Width-Modulated Computer-Controlled Systems*, J. Franklin Inst. 277, No. 2, 128-139 (Feb. 1964).

G. J. Murphy and T. Meksawan, *Time-Optimal Pulse-Width Control of Linear Systems*, Proc. NEC, 20, 596-600 (1964).

W. J. Murphy, *Optimal Adaptive Control of Linear Discrete Systems*, Ph.D. Thesis, Washington Univ., 1967. (UM No. 67-9390).

G. E. Myers, *Properties of the Conjugate Gradient and Davidon Methods, Part 5*, NASA-CR-92127 (Apr. 1968).

J. Nagumo and A. Noda, *A Learning Method for System Identification*, Proc. NEC, 22, 584-589 (1966).

N. E. Nahi, *Optimum Control of Linear Systems with a Modified Energy Constraint*, IEEE Trans. Automat. Contr., AC-9, No. 2, 137-143 (Apr. 1964).

N. E. Nahi and L. A. Wheeler, *An Iterative Procedure for Solving the Discrete Terminal Control Problem*, Proc. NEC, 22, 671-676 (1966).

K. S. Narendra and Y. S. Cho, *Stability Analysis of Nonlinear and Time-Varying Discrete Systems*, Proc. Hawaii Int. Conf. Syst. SCI (Honolulu, Jan. 1968), 166-169.

K. S. Narendra and I. J. Shapiro, *On Two Time-Scales in Linear Systems with Special Reference to Adaptive Control*, Proc. JACC, 422-429 (1968).

S. Narita et al, *Optimal System Operation by the Discrete Maximum Principle*, IEEE Conf. Rec., Power Industry Computer Appl. (Pittsburgh, Pa., May 1967), 189-207.

SELECTED CONTROL BIBLIOGRAPHY

T. H. Naylor et al, *Computer Simulation Techniques* (New York: John Wiley & Sons, Inc., 1966).

S. R. Neal, *Linear Estimation in the Presence of Errors in Assumed Plant Dynamics*, IEEE Trans. Automat. Contr., AC-12, No. 5, 592-594 (Oct. 1967).

W. L. Nelson, *Application of Optimal Control and Estimation Theory to Missile Guidance*, Stochastic Optimization and Control (ed.) H. F. Karreman (New York: John Wiley & Sons, Inc., 1968), 133-171.

G. L. Nemhauser, *Introduction to Dynamic Programming* (New York: John Wiley & Sons, Inc., 1966).

L. W. Neustadt, *Discrete Time Optimal Control Systems*, Proc. Int. Symp. Nonlinear Differential Equations, Nonlinear Mechanics (ed.) J. P. LaSalle (New York: Academic Press, 1963), 267-283.

L. W. Neustadt and B. H. Paiewonsky, *On Synthesizing Optimal Controls*, Proc. IFAC, 2, 283-291 (1963).

P. M. Newbold and J. H. Westcott, *The Application of Bayesian Statistics to the Control of Noisy Extremum Systems*, Proc. NEC, 22, 614-619 (1966).

H. Nicholson, *Eigenvalue and State-Transition Sensitivity of Linear Systems*, Proc. Inst. Elec. Eng., 114, No. 12, 1991-1995 (Dec. 1967).

C. E. Nickel, *Linear Error Analysis and Filtering by Recursive Least Squares*, HSM-M58-67 (June 1967).

R. A. Nieman, *Variable Time Optimal Control*, NASA-CR-84510 (Apr. 1967).

Z. J. Nikolic and K. S. Fu, *A Mathematical Model of Learning in an Unknown Random Environment*, Proc. NEC, 22, 607-612 (1966).

T. Nishimura, *On the A-Priori Information in Sequential Estimation Problems*, Proc. NEC, 21, 511-516 (1965).

V. W. Noonburg, *A Nonlinear System of Differential-Difference Equations*, Ph.D. Thesis, Cornell Univ., 1967 (UM No. 68-887).

A. R. M. Noton, *Introduction to Variational Methods in Control Engineering* (New York: Pergamon Press, 1965).

A. R. M. Noton, *Optimal Control and the Two-Point Boundary Problem*, Proc. Inst. Mech. Eng., 179, Pt. 3H, 181-188 (1964-65).

A. R. M. Noton, P. Dyer, and C. A. Markland, *Numerical Computation of Optimal Control*, IEEE Trans. Automat. Contr., AC-12, No. 1, 59-66 (Feb. 1967).

SELECTED CONTROL BIBLIOGRAPHY

V. N. Novoseltsev, *Prediction of Optimal Control Sequence*, Theory of Self-Adaptive Control Systems (ed.) P. H. Hammond (New York: Plenum Press, 1966), 28-31.

J. P. O'Donohue, *Transfer Function for a Stepper Motor*, Contr. Eng., 8, No. 11, 103-104 (Nov. 1961).

K. Okamura, *Some Mathematical Theory of the Penalty Method for Solving Optimum Control Problems*, SIAM J. Contr., 2, No. 3, 317-331 (1965).

R. Oldenburger, *Optimum Nonlinear Control*, Trans. ASME, 79, No. 3, 527 (Apr. 1957).

R. Oldenburger (ed), *Optimal and Self-Optimizing Control* (Cambridge, Mass.: M.I.T. Press, 1966).

R. Oldenburger, *Optimal Control* (New York: Holt, Rinehart, and Winston, 1966).

R. Oldenburger and Y. Ikebe, *Linearization of Time-Independent Nonlinearities by Use of an Extra Signal and Extra Nonlinearity*, J. Basic Eng., 89, No. 2, 241-250 (June 1967).

A. V. Oppenheim, *Superposition in a Class of Nonlinear Systems*, NASA-CR-63063 (May 1964).

A. V. Oppenheim, *Superposition in a Class of Nonlinear Systems*, IEEE Int. Conv. Rec., Pt. I, 171-177 (1964).

J. M. Ortega and M. L. Rockoff, *Nonlinear Difference Equations and Gauss-Seidel Type Iterative Methods*, SIAM J. Numer. Anal., 3, No. 2, 497-513 (1966).

R. P. O'Shea, *The Extension of Zubov's Method to Sampled Data Control Systems Described by Nonlinear Autonomous Difference Equations*, IEEE Trans. Automat. Contr., AC-9, No. 1, 62-70 (Jan. 1964).

R. P. O'Shea, *Approximation of the Asymptotic Stability Boundary of Discrete-Time Control Systems Using an Inverse Transformation Approach*, IEEE Trans. Automat. Contr., AC-9, No. 4, 441-448 (Oct. 1964).

K. Oza, *Identification Problem and Random Contraction Mappings*, Ph.D. Thesis, Univ. California, 1967. (UM No. 68-10390).

M. A. Pai, *The Analysis of Nonlinear Feedback Sampled-Data Systems*, AFOSR-1791 (July 1961).

B. Paiewonsky, *Optimal Control: A Review of Theory and Practice*, AIAA J., 3, No. 11, 1985-2006 (Nov. 1965).

SELECTED CONTROL BIBLIOGRAPHY

G. Paine, *The Application of the Method of Quasilinearization to the Computation of Optimal Control*, NASA-CR-90749 (Aug. 1967).

R. Pallu de la Barriere, *Optimal Control Theory* (Philadelphia: W. B. Saunders Co., 1967).

J. Pappas, *Application of the Kalman Filter to Sequential Parameter Estimation via Householder's Matrix Inversion Method*, RO-S-67-1 (June 1967).

P. C. Parks and B. L. Clarkson, *Some Theory and Practice of Random Process Analysis*, Proc. Inst. Mech. Eng., 179, Pt. 3H, 105-115 (1964-65).

A. E. Pearson, *On Adaptive Optimal Control of Nonlinear Processes*, J. Basic Eng., 86, No. 1, 151-159 (Mar. 1964).

A. E. Pearson, *Adaptive Optimal Steady State Control of Nonlinear Systems*, Advances in Control Systems, 5,(ed.) C. T. Leondes (New York: Academic Press, 1967), 1-50.

J. B. Pearson, Jr. and J. E. Gibson, *On the Asymptotic Stability of a Class of Saturating Sampled-Data Systems*, IEEE Trans. Appl. Ind., 83, No. 71, 81-86 (Mar. 1964).

J. B. Pearson, Jr., *A Note on the Stability of a Class of Optimum Sampled-Data Systems*, IEEE Trans. Automat. Contr., AC-10, No. 1, 117 (Jan. 1965).

J. B. Pearson, Jr. and R. Sridhar, *A Discrete Optimal Control Problem*, IEEE Trans. Automat. Contr., AC-11, No. 2, 171-174 (Apr. 1966).

J. B. Pearson, Jr. and R. Sridhar, *A Discrete Maximum Principle via Convex Programming*, Proc. IFAC, 1, Bk. 2, Paper No. 18F (1966).

J. B. Pearson, Jr., *A Note on Nonlinear Filtering*, IEEE Trans. Automat. Contr., AC-13, No. 1, 103-105 (Feb. 1968).

J. D. Pearson, *Approximation Methods in Optimal Control; I: Sub-Optimal Control*, J. Electron. Contr., 13, No. 5, 453-469 (Nov 1962).

J. D. Pearson, *Reciprocity and Duality in Control Programming Problems*, J. Math. Anal. Appl., 10, No. 2, 388-408 (Apr. 1965).

J. D. Pearson, *The Discrete Maximum Principle*, Int. J. Contr., Ser. 1, 2, No. 2, 117-124 (Aug. 1965).

J. D. Pearson, *On the Duality Between Estimation and Control*, SIAM J. Contr., 4, No. 4, 594-600 (1966).

SELECTED CONTROL BIBLIOGRAPHY

E. E. Pentecost and A. R. Stubberud, *Synthesis of Computationally Efficient Sequential Linear Estimators*, IEEE Trans. Aerospace Electron. Syst., AES-3, No. 2, 242-249 (Mar. 1967).

V. L. Pereyra, *Iterated Deferred Corrections for Nonlinear Operator Equations*, MRC-TSR-763 (July 1967).

V. L. Pereyra, *Highly Accurate Discrete Methods for Nonlinear Problems*, Ph.D. Thesis, Univ. Wisconsin, 1967. (UM No. 67-9016).

V. L. Pereyra, *Iterative Methods for Solving Nonlinear Least Squares Problems*, SIAM J. Numer. Anal., 4, No. 1, 27-36 (1967).

A. A. Pervozvanskii, *Random Processes in Nonlinear Control Systems* (New York: Academic Press, 1965).

J. Peschon (ed.), *Disciplines and Techniques of Systems Control* (New York: Blaisdell Publishing Co., 1965).

J. Peschon and L. Meier, *Optimum Discrete Information Systems*, Proc. JACC, 265-271 (1966).

V. Peterka, *Combination of Finite Settling Time and Minimum Integral of Squared Error in Digital Control Systems*, Proc. IFAC, 2, 196-203 (1963).

E. L. Peterson, *Statistical Analysis and Optimization of Systems* (New York: John Wiley & Sons, Inc., 1961).

P. Petrov, *Variational Methods in Optimum Control Theory* (New York: Academic Press, 1968).

S. C. Pincura, *A Stability Criterion for Certain Multiplicative Nonlinear Control Systems*, Proc. JACC, 787-796 (1968).

B. G. Pittel, *An Optimal Control Problem Connected with the Minimization of a Functional of the "Maximum Deviation" Type*, Differential Equations, 1, No. 11, 1173-1186 (Nov. 1965).

J. B. Plant, *Some Iterative Solutions in Optimal Control* (Cambridge, Mass.: M.I.T. Press, 1967).

R. N. A. Plimmer, *Theoretical and Computational Aspects of Optimum Control*, RAE-TR-67320 (Apr. 1966).

E. Polak, *Minimal Time Control of a Discrete System with a Nonlinear Plant*, IEEE Trans. Automat. Contr., AC-8, No. 1, 49-57 (Jan. 1963).

L. S. Pontryagin et al, *The Mathematical Theory of Optimal Processes* (New York: MacMillan Co., 1964).

SELECTED CONTROL BIBLIOGRAPHY

J. C. Pope, *Adaptive Control System with Model Reference*, Ph.D. Thesis, Univ. California, 1966. (UM No. 66-8360).

E. P. Popov, *Certain Problems of the Synthesis of Non-linear Systems of Automatic Control*, Proc. IFAC, 1, 165-172 (1960).

V. M. Popov, *Criterion of Quality for Non-linear Controlled Systems*, Proc. IFAC, 1, 173-246 (1960).

G. Porcelli and K. A. Fegley, *Linear Programming Design of Digitally Compensated Systems*, Proc. JACC, 412-421 (1964).

G. Porcelli and K. A. Fegley, *Optimal Design of Digitally Compensated Systems by Quadratic Programming*, J. Franklin Inst., 282, No. 5, 303-317 (Nov. 1966).

W. A. Porter, *Modern Foundations of Systems Engineering* (New York: MacMillan Co., 1966).

J. E. Potter, *Matrix Quadratic Solutions*, J. Soc. Ind. Appl. Math., 14, No. 3, 496-502 (May 1966).

C. Pottle, *The Digital Adaptive Control of a Linear Process Modulated by Random Noise*, Univ. Illinois, Coordinated Sci. Lab. Rep. No. R-134 (Feb. 1962).

C. Pottle, *The Digital Adaptive Control of a Linear Process Modulated by Random Noise*, IEEE Trans. Automat. Contr., AC-8, No. 3, 228-234 (July 1963).

R. S. Printiss, *Estimation of Unknown Probability Density Functions from Observed Data*, SC-DC-67-2118 (Sept. 1967).

A. I. Propoi, *The Maximum Principle for Discrete Control Systems*, Automat. Remote Contr., 26, No. 7, 1167-1177 (July 1965).

R. F. Prueher, *An Equivalent Gain and Stochastic Analysis for Nonlinear Sampled-Data Control Systems*, AFIT-TR-67-1 (1966).

V. S. Pugachev, *Non-linear System Theory*, Proc. IFAC, 2, 1-13 (1963).

V. S. Pugachev, *Theory of Random Functions and Its Application to Control Problems* (Reading, Mass.: Addison-Wesley Publishing Co., 1965).

N. N. Puri, *Optimal Design of Regulators*, J. Franklin Inst., 281, No. 6, 499-513 (June 1966).

B. M. Rahm, *Discrete Linear Optimal Control System Synthesis with Quadratic Cost Functionals*, Proc. 20th Annual Southwestern IEEE Conf. (Houston, Texas, Apr. 1968), Paper No. 15B.

SELECTED CONTROL BIBLIOGRAPHY

N. S. Raibman and A. T. Terekhin, *Variance-Ratio Methods for Random Processes and Their Application to the Analysis of Nonlinear Control Plants*, Automat. Remote Contr., 26, No. 3, 496-506 (Mar. 1965).

H. E. Rausch, F. Tung, and C. T. Striebel, *Maximum Likelihood Estimates of Linear Dynamic Systems*, AIAA J., 3, No. 8, 1445-1450 (Aug. 1965).

R. Reiss et al., *Statistical Control System Design*, AFFDL-TR-66-83 (Oct. 1966).

Z. V. Rekasius, *Optimal Linear Regulators with Incomplete State Feedback*, IEEE Trans. Automat. Contr., AC-12, No. 3, 296-299 (June 1967).

M. G. Rekoff, *State Variables in Sampled-Data Systems*, Control, 11, 535-538 (Nov. 1967).

K. L. Remmler et al., *Solutions of Systems of Nonlinear Equations*, NASA-CR-84448 (Oct. 1966).

A. M. Revington and J. C. Hung, *Time Weighted Energy Control of Discrete-Data Control Systems*, IEEE Trans. Automat. Contr., AC-11, No. 4, 758-759 (Oct. 1966).

P. A. Reynolds, *Amplitude-Limited Control of Linear, Discrete Systems with Quadratic Error Measure*, AD 622 513 (Sept. 1965).

F. M. Reza, *Functions of a Matrix*, RADC-TR-67-376 (Nov. 1967).

S. O. Rice, *Mathematical Analysis of Randon Noise*, Bell Syst. Tech. J., 23, 282-332 (July, 1944).

J. H. Rillings and R. J. Roy, *Control of a Nonlinear Plant with Two Level Input Using a Tabular Adaptive-Predictive Model*, Proc. NEC, 22, 540-545 (1966).

R. E. Rink, *Controllability and Optimal Control of Bilinear Systems*, Ph.D. Thesis, Univ. New Mexico, 1967. (UM No. 68-3479).

R. E. Rink and R. R. Mohler, *Completely Controllable Bilinear Systems*, SIAM J. Contr., 6, No. 3, 477-486 (Aug. 1968).

H. M. Robbins, *A Generalized Legendre-Clebsch Condition for the Singular Cases of Optimal Control*, IBM J. Res. Develop., 11, No. 4, 361-372 (July 1967).

S. M. Roberts, *Dynamic Programming in Chemical Engineering and Process Control* (New York: Academic Press, 1964).

S. M. Roberts, *Dynamic Programming and Lagrangian Multipliers*, Ind. Eng. Chem. Fundamentals, 4, No. 4, 488-490 (Nov. 1965).

SELECTED CONTROL BIBLIOGRAPHY

S. M. Roberts and J. S. Shipman, *Some Results in Two-Point Boundary Value Problems*, IBM J. Res. Develop., 11, No. 4, 383-388 (July 1967).

A. S. Robinson, *Control Programming - Key to the Synthesis of Efficient Digital Computer Control Systems*, AIEE Trans. Appl. Inc., 79, No. 52, 475-502 (Jan. 1961).

S. M. Robinson, *Interpolative Solution of Systems of Nonlinear Equations*, SIAM J. Numer. Anal., 3, No. 4, 650-658 (1966).

K. M. Roehr, *Limits in Real-Time Digital Filtering of Non-stationary Random Processes*, Ph.D. Thesis, George Washington Univ., 1967. (U.M. No. 67-8827).

J. G. Root, *Optimum Control of Linear, Sampled Data Systems with Inaccessible State Variables*, RM-5655-PR (May 1968).

J. B. Rosen, *Sufficient Conditions for Optimal Control of Convex Processes*, PB-176 753 (May 1964).

J. B. Rosen, *Optimal Control and Convex Programming*, Univ. Wisconsin, Math. Res. Center Tech. Rep. No. 547 (1965).

J. B. Rosen, *Iterative Solution of Nonlinear Optimal Control Problems*, SIAM J. Contr., 4, No. 1, 223-244 (Feb. 1966).

J. B. Rosen, *The Steepest Ascent/Descent Method*, NYO-2262TA-157 (May 1967).

B. F. Rothenberger and L. Lapidus, *The Control of Nonlinear Systems: Part 3, Invariant Imbedding and Quasilinearization*, A.I.Ch.E.J., 13, No. 1, 114-117 (Jan. 1967).

B. F. Rothenberger and L. Lapidus, *The Control of Nonlinear Systems. IV. Quasilinearization as a Numerical Method*, A.I.Ch.E.J.,13, No. 5, 973-981 (Sept. 1967).

B. F. Rothenberger and L. Lapidus, *The Control of Nonlinear Systems. V. Quasilinearization and State-Constrained Systems*, A.I.Ch.E.J., 13, No. 5, 982-988 (Sept. 1967).

Yu. A. Rozanov, *Stationary Random Processes* (San Francisco: Holden-Day, 1967).

L. I. Rozonoer, *L. S. Pontryagin's Maximum Principle in the Theory of Optimum Systems*, Automat. Remote Contr., 20, 1288-1302, 1405-1421, 1517-1532 (1959).

H. Rund, *The Hamilton-Jacobi Theory in the Calculus of Variations* (London: D. Van Nostrand Co. Ltd., 1966).

SELECTED CONTROL BIBLIOGRAPHY

D. L. Russell, *Penalty Functions and Bounded Phase Co-ordinate Control*, SIAM J. Contr., 2, No. 3, 409-422 (1965).

H. Sagan, *Dynamic Programming and Pontryagin's Maximum Principle*, NASA-CR-838 (July 1967).

A. P. Sage and W. C. Choate, *Minimum Time Identification of Non-stationary Dynamic Processes*, Proc. NEC, 21, 587-592 (1965).

A. P. Sage and B. R. Eisenberg, *Suboptimal Adaptive Control of a Nonlinear Plant*, IEEE Trans. Automat. Contr., AC-11, No. 3, 621-623 (July 1966).

A. P. Sage and G. W. Masters, *On-Line Estimation of States and Parameters for Discrete Nonlinear Dynamic Systems*, Proc. NEC, 22, 677-682 (1966).

A. P. Sage, *Invariant Imbedding and the Computational Solution of Optimization and Estimation Problems*, Proc. 20th Annual Southwestern IEEE Conf. (Houston, Texas, Apr. 1968), Paper No. 15C.

Y. Sakawa and C. Hayashi, *Solution of Optimum Control Problems by Using Pontryagin's Maximum Principle*, Proc. IFAC, 2, 339-346 (1963).

I. W. Sandberg, *Frequency-Domain Criteria for the Stability of Nonlinear Feedback Systems*, Proc. NEC, 20, 737-741 (1964).

P. E. Sarachik and G. M. Kranc, *On Optimal Control of Systems with Multi-norm Constraints*, Proc. IFAC, 2, 306-314 (1963).

P. E. Sarachik, *An Approach to the Design of Nonlinear Discrete Self-Optimizing Systems*, Proc. NEC, 20, 617-619 (1964).

P. E. Sarachik and E. Kreindler, *Controllability and Observability of Linear Discrete-Time Systems*, Int. J. Contr., 1, No. 5, 419-432 (May 1965).

G. N. Saridis, *On the Exact and Approximate Solutions of the Optimal Control Problem with Bounded State Variables*, Ph.D. Thesis, Purdue Univ., 1965. (UM No. 66-5298).

V. A. Sastry and M. D. Srinath, *Sub-optimal Policies for Two Classes of Control*, Int. J. Contr., 3, No. 6, 497-512 (June 1966).

E. S. Savas, *Computer Control of Industrial Processes* (New York: McGraw-Hill Book Co., 1965).

Y. Sawaragi, Y. Sunahara, and K. Inoue, *Near-Optimal Control of Nonlinear Dynamic Plants with a Quadratic Performance Index*, Mem. Fac. Engrg. Kyoto Univ., 28, Pt. 1, 152-163 (Jan. 1966).

SELECTED CONTROL BIBLIOGRAPHY

Y. Sawaragi and T. Katayama, *On the Estimation of State Variables for Noisy Discrete-Time Nonlinear Dynamical Systems*, Kyoto Univ., Eng. Res. Inst. Rep. No. 132 (Mar. 1967).

Y. Sawaragi, Y. Sunahara, and T. Nakamizo, *Statistical Decision Theory in Adaptive Control Systems* (New York: Academic Press, 1967).

L. L. Scalzott and C. F. Lorenzo, *Practical Stability Criterion and Its Application to Digital Simulation*, NASA-TN-D-4203 (Dec. 1967).

R. S. Schechter, *The Variational Method in Engineering* (New York: McGraw-Hill Book Co., 1967).

F. H. Schlee, C. J. Standish, and N. F. Toda, *Divergence in the Kalman Filter*, AIAA J., 5, No. 6, 1114-1120 (June 1967).

C. H. Schley, *Optimal Control for the Engineer; II: Dynamic Optimization Program (DYNOP)*, 67-C-130 (Apr. 1967).

S. F. Schmidt, *Estimation of State with Acceptable Accuracy Constraints*, NASA-CR-84795 (Jan. 1967).

H. Schneider and G. P. Barker, *Matrices and Linear Algebra* (New York: Holt Rinehart and Winston, Inc., 1968).

J. M. Schuler, C. R. Chalk, and A. E. Schelhorn, *Application and Evaluation of Certain Adaptive Control Techniques in Advanced Flight Vehicles. Vol. I. G.E. Self-adaptive Flight Control System*, ASD-TR-61-104 (July 1961).

D. G. Schultz, *Control System Design by State Variable Feedback Techniques*, NASA-CR-77901 (July 1966).

D. G. Schultz and J. L. Melsa, *State Functions and Linear Control Systems* (New York: McGraw-Hill Book Co., 1967).

L. Schwartz, *Approximate Continuous Nonlinear Minimal-Variance Filtering*, NASA-CR-84517 (Apr. 1967). Ph.D. Thesis, Univ. California, 1966. (UM No. 67-6189).

L. Schwartz and E. B. Stear, *A Valid Mathematical Model for Approximate Nonlinear Minimal-Variance Filtering*, J. Math. Anal. Appl., 21, No. 1, 1-6 (Jan. 1968).

L. Schwartz and E. B. Stear, *A Computational Comparison of Several Nonlinear Filters*, IEEE Trans. Automat. Contr., AC-13, No. 1, 83-86 (Feb. 1968).

A. K. Sen, *The Use of Complex Equivalent Gain of a Memory-Type Nonlinearity for the Analysis of Feedback Control Systems with Random Inputs*, IEEE Trans. Automat. Contr., AC-9, No. 3, 245-248 (July 1964).

SELECTED CONTROL BIBLIOGRAPHY

S. Shapiro, *Lagrange and Mayer Problems in Optimal Control*, Automatica, 3, Nos. 3/4, 219-230 (Jan. 1966).

L. Shaw, *Research in Control Theory*, PIBEE-68-0001 (Mar. 1968).

J. C. Shellenbarger, *Estimation of Covariance Parameters for an Adaptive Kalman Filter*, Ph.D. Thesis, Iowa State Univ. of Science and Technology, 1966. (UM No. 66-10, 439).

J. C. Shellenbarger, *Estimation of Covariance Parameters for an Adaptive Kalman Filter*, Proc. NEC, 22, 698-702 (1966).

J. C. Shellenbarger, *A Multivariance Learning Technique for Improved Dynamic System Performance*, Proc. NEC, 23, 146-151 (1967).

J. E. Shemer and S. C. Gupta, *Applications of Butkovskii's Form of Discrete Maximum Principle*, ISA Trans., 5, No. 4, 395-405 (Oct. 1966).

S. M. Shinners, *Optimal and Adaptive Control Systems*, Electro-Technology, 74, No. 1, 63-80 (July 1964).

R. Sivan, *An Extension of the Best Linear Controller to a Polynomial Controller for Non-Gaussian Disturbances*, IEEE Trans. Automat. Contr., AC-9, No. 3, 296-297 (July 1964).

R. Sivan, *The Necessary and Sufficient Conditions for the Optimal Controller to be Linear*, Proc. JACC, 297-304 (1964).

L. J. Skidmore, *Random Processes*, AD-484382 (Apr. 1965).

J. B. Slaughter, *Quantization Errors in Digital Control Systems*, IEEE Trans. Automat. Contr., AC-9, No. 1, 70-74 (Jan. 1964).

G. A. Smith, *The Theory and Application of Least Squares*, NASA-TM-X-63127 (Dec. 1967).

G. L. Smith, S. F. Schmidt, and L. A. McGee, *Application of Statistical Filter Theory to the Optimal Estimation of Position and Velocity on Board a Circumlunar Vehicle*, NASA-TR-R-135 (1962).

H. W. Smith, *Approximate Analysis of Randomly Excited Nonlinear Controls* (Cambridge, Mass.: M.I.T. Press, 1966).

O. J. M. Smith, *Deadbeat Sampled System Direct Synthesis*, IEEE Trans. Automat Contr., AC-8, No. 3, 240-246 (July 1963).

S. L. Smith, *Model and Identification Theory for Discrete Systems*, Ph.D. Thesis, Univ. Florida, 1966. (UM No. 67-3510).

J. I. Soliman and A. Al-Shaikh, *General Performance Indices for the Time and Frequency Response for the Free Motion of Linear Discrete Control Systems*, J. Mech. Eng. Sci., 7, No. 4, 424-430 (1964).

SELECTED CONTROL BIBLIOGRAPHY

J. I. Soliman and A. Al-Shaikh, *Weighted Performance Criteria for the Synthesis of Discrete Systems*, IEEE Trans. Automat. Contr., AC-11, No. 2, 277-281 (Apr. 1966).

J. I. Soliman and A. Al-Shaikh, *A Useful Relationship for Studying the Effect of a Single Variable Parameter in Discrete Systems*, IEEE Trans. Automat. Contr., AC-11, No. 4, 759 (Oct. 1966).

J. I. Soliman and A. Al-Shaikh, *Performance Criteria for the Free and Forced Motion of Discrete Control Systems*, Proc. Inst. Elec. Eng., 113, No. 11, 1902-1906 (Nov. 1966).

T. T. Soong, *On A Priori Statistics in Minimum Variance Estimation Problems*, Proc. JACC, 388-391 (1964).

H. W. Sorenson and A. R. Stubberud, *Recursive Filtering for Systems with Small but Non-negligible Non-linearities*, Int. J. Contr., First Series, 7, 281-298 (Mar. 1968).

H. W. Sorenson, *On the Error Behavior in Linear Minimum Variance Estimation Problems*, IEEE Trans. Automat. Contr., AC-12, No. 5, 557-562 (Oct. 1967).

H. W. Sorenson, *Controllability and Observability of Linear, Stochastic, Time-Discrete Control Systems*, Advances in Control Systems, 6, (ed.) C. T. Leondes (New York: Academic Press, 1968), 95-158.

H. A. Spang, *Optimum Control of an Unknown Linear Plant Using Bayesian Estimation of the Error*, Proc. NEC, 20, 620-625 (1964).

R. Sridhar et al., *Application of Modern Control and Nonlinear Estimation Techniques*, NASA-CR-91441 (Dec. 1967).

H. J. Stetter, W. Baron, and H. Sochatzy, *Stability Regions of Discrete Variable Methods for Ordinary Differential Equations*, AFOSR-67-2329 (Sept. 1967).

R. L. Stratonovich, *Most Recent Development of Dynamic Programming Techniques and Their Application to Optimal Systems Design*, Proc. IFAC, 2, 352-357 (1963).

R. L. Stratonovich, *Conditional Markov Processes and Their Application to the Theory of Optimal Control* (New York: American Elsevier Publishing Co., Inc., 1968).

A. Strauss, *An Introduction to Optimal Control Theory* (New York: Springer-Verlag, 1968).

J. C. Strauss and W. L. Gilbert, *SCADS: A Programming System for the Simulation of Combined Analog Digital Systems*, AFOSR-67-2518 (Mar. 1964).

SELECTED CONTROL BIBLIOGRAPHY

J. G. Sullivan, *The Effect on Program Execution Time of the Use of FORTRAN Multisubscripted Variables*, ORNL-TM-1969 (Sept. 1967).

A. Sumner, *A Method of Calculating the Generalized Performance Index of Linear Systems*, AEEW-M583 (Oct. 1965).

H. M. Sumner, *FIFI 3: A Digital Computer Code for the Solution of Sets of First Order Differential Equations and the Analysis of Process Plant Dynamics*, AEEW-R453 (Nov. 1965).

Y. Sunahara, *An Approximation Method of State Estimation for Nonlinear Dynamical Systems*, Brown Univ., Div. Appl. Math. Tech. Rep. No. 67-8 (Dec. 1967).

P. Swerling, *First Order Error Propagation in a Stagewise Smoothing Procedure for Satellite Observations*, J. Astronaut Sci., 6, No. 3, 46-52 (Autumn 1959).

P. Swerling, *Topics in Generalized Least Squares Signal Estimation*, P-3007-1 (Sept. 1965).

D. D. Sworder, *On the Structural Properties of Some Discrete Time Adaptive Control Systems*, Proc. NEC, 20, 612-616 (1964).

D. D. Sworder, *Optimal Control of Discrete-Time Stochastic Systems*, J. Math. Anal. Appl., 15, No. 2, 253-263 (Aug. 1966).

D. D. Sworder, *Optimal Adaptive Control Systems* (New York: Academic Press, 1966).

D. D. Sworder, *An Inverse Problem in Discrete-Time Adaptive Control*, Proc. NEC, 22, 546-550 (1966).

D. D. Sworder, *Control of a Linear Discrete-Time Stochastic System with a Bounded Input*, Proc. JACC, 450-456 (1966).

G. P. Szego and J. B. Pearson, Jr., *On the Absolute Stability of Sampled-Data Systems: The Indirect Control Case*, IEEE Trans. Automat. Contr., AC-9, No. 2, 160-163 (Apr. 1964).

D. Tabak and B. C. Kuo, *Determination of the Output Statistics of a Sampled-Data System with a Nonlinearity*, IEEE Int. Conv. Rec., 14, Pt. 6, 173-178 (Mar. 1966).

M. B. Tamburro, A. S. Abbott, and G. E. Townsend, *Guidance, Flight Mechanics and Trajectory Optimization. Vol. 1: Coordinate Systems and Time Measurement*, NASA-CR-1000 (Feb. 1968).

C. F. Taylor, *Synthesis of a Nonlinear Control System*, Ph.D. Thesis, Stanford Univ., 1956. (UM No. 16,033).

SELECTED CONTROL BIBLIOGRAPHY

G. J. Thaler and M. P. Pastel, *Analysis and Design of Nonlinear Feedback Control Systems* (New York: McGraw-Hill Book Co., 1962).

J. G. Thompson and R. H. Kohr, *Modeling and Compensation of Nonlinear Systems Using Sensitivity Analysis*, Proc. JACC, 505-518 (1967).

V. I. Toloknov, *Analytical Design of the Optimal Regulator*, Eng. Cybern., No. 5, 145-149 (Sept.-Oct. 1966).

R. Tomovic, *Introduction to Nonlinear Automatic Control Systems* (New York: John Wiley & Sons, Inc., 1966).

H. C. Torng, *Optimization of Discrete Control Systems through Linear Programming*, J. Franklin Inst., 278, No. 1, 28-44 (July 1964).

H. L. Torrey, G. J. Farris, and L. E. Burkhart, *DIAN II: An All-FORTRAN System for Programming a Digital Computer in Terms of Analog Components*, IS-1656 (Aug. 1967).

J. T. Tou and J. B. Lewis, *A Study of Nonlinear Digital Control Systems*, Purdue Univ. Tech. Rep. No. 201 (Aug. 1961).

J. T. Tou, *Digital and Sampled Data Control Systems* (New York: McGraw-Hill Book Co., 1959).

J. T. Tou, P. D. Joseph, and J. B. Lewis, *A Study of Digital Adaptive Control Systems*, Purdue Univ. Tech. Rep. No. 1 (Sept. 1960).

J. T. Tou, *Statistical Design of Linear Discrete-Data Control Systems via the Modified Z-Transform Method*, J. Franklin Inst., 271, No. 4, 249-262 (Apr. 1961).

J. T. Tou, *Design of Optimum Digital Control Systems via Dynamic Programming*, Proc. Dynamic Programming Workshop (Boulder, Colo., June, 1961), 37-66.

J. T. Tou and P. D. Joseph, *Optimum Design of Linear Multivariable Digital Control Systems*, Purdue Univ. Tech. Rep. No. 102 (Aug. 1961).

J. T. Tou and B. Vadhanaphuti, *Optimum Control of Nonlinear Discrete-Data Systems*, AIEE Trans. Appl. Ind., 80, No. 56, 166-171 (Sept. 1961).

J. T. Tou and P. D. Joseph, *Modern Synthesis of Computer Control Systems*, IEEE Trans. Appl. Ind., 82, No. 66, 61-65 (May 1963).

J. T. Tou, *Synthesis of Discrete Systems Subject to Control-Signal Saturation*, J. Franklin Inst., 277, No. 5, 401-413 (May 1964).

J. T. Tou and R. H. Wilcox (eds.), *Computer and Information Sciences* (Wash., D. C.: Spartan Books, Inc., 1964).

SELECTED CONTROL BIBLIOGRAPHY

J. T. Tou, *Solution of an Optimization Problem for Linear Discrete Systems through Ordinary Calculus*, IEEE Trans. Automat. Contr., AC-10, No. 2, 209-211 (Apr. 1965).

J. T. Tou, *System Optimization via Learning and Adaptation*, Int. J. Contr., 2, No. 1, 21-31 (July 1965).

J. T. Tou, *Dynamic Programming and Modern Control Theory*, Progr. Contr. Eng., 3, 1-34 (1966).

J. T. Tou and H. H. Yeh, *An Extension of Pontryagin's Maximum Principle*, Ohio State Univ., Res. Foundation (Sept. 1967).

J. G. Truxal, *Adaptive Control*, Proc. IFAC, 2, 386-392 (1963).

Y. A. Tsypkin, *Optimal Processes in Sampled Data Control Systems*, STL-TR-61-5110-27 (July 1961).

Y. Z. Tsypkin, *Fundamentals of the Theory of Non-linear Pulse Control Systems*, Proc. IFAC, 2, 172-180 (1963).

Y. Z. Tsypkin, *Theory of Linear Pulse Systems*, FTD-MT-64-133 (June 1966).

W. G. Tuel, *An Improved Algorithm for the Solution of Discrete Regulation Problems*, IEEE Trans. Automat. Contr., AC-12, No. 5, 522-528 (Oct. 1967).

W. G. Tuel, *A Simplified Algorithm for the Solution of the Steady State Discrete Matrix Riccati Equation*, Proc. JACC, 549-556 (1967).

F. Tung and C. T. Striebel, *A Stochastic Optimal Control Problem and Its Applications*, J. Math. Anal. Appl., 12, No. 2, 350-359 (Oct. 1965).

M. Tyner and F. P. May, *Process Engineering Control* (New York: Ronald Press Co., 1968).

P. E. Uhlrich, *Analysis of Nonlinear Sampled Data Systems by Finite-Difference Techniques*, Ph.D. Thesis, Univ. Wisconsin, 1964. (UM No. 64-10 328).

H. L. Van Trees, *Synthesis of Optimum Nonlinear Control Systems* (Cambridge, Mass.: M.I.T. Press, 1962).

A. B. Vasileva, *Asymptotic Behaviour of Solutions to Certain Problems Involving Non-linear Differential Equations Containing a Small Parameter Multiplying the Highest Derivative*, Russian Math. Surveys, 18, No. 3, 13-84 (May-June 1963).

L. N. Volgin, *Synthesis of Optimum Sampled-Data Systems*, Proc. IFAC, 2, 181-185 (1963).

SELECTED CONTROL BIBLIOGRAPHY

A. V. Vulifson, *Numerical Digital Computer Method for Determining the Transient Responses of Nonlinear Automatic Systems Based on Calculation of the Convolution Integral*, NASA-TM-X-60554 (Aug. 1967).

L. Wegge, *On a Discrete Version of the Newton-Raphson Method*, SIAM J. Numer. Anal., 3, No. 1, 134-142 (1966).

S. Wegrzyn and P. Vidal, *On the Absolute Stability of Continuous and Sampled-Data Nonlinear Systems*, Compt. Rend. Acad. Sci. (Paris), Ser. A, 266, No. 26, 1297-1299 (June 1968). (In French).

J. H. Wescott, J. J. Florentin, and J. D. Pearson, *Approximation Methods in Optimal and Adaptive Control*, Proc. IFAC, 2, 263-273 (1963).

J. H. Wescott, *Application of Optimal Methods to Control of Industrial Processes*, Proc. IBM Sci. Comput. Symp. Contr. Theory Appl. (Yorktown Heights, N. Y., Oct. 1964), 89-102 (1966).

J. C. West, *Analytical Techniques for Nonlinear Control Systems* (Princeton, N. J.: D. Van Nostrand Co., Inc., 1961).

J. H. Westcott et al., *Optimal Techniques for On-line Control*, Proc. IFAC, 1, Bk. 2, Paper No. 29B (1966).

H. P. Whitaker, *Design Capabilities of Model Reference Adaptive Systems*, Proc. NEC, 18, 241-249 (1962).

P. Whittle, *Prediction and Regulation by Linear Least-Square Methods* (Princeton, N. J., D. Van Nostrand Co., Inc., 1963).

W. S. Widnall, *Applications of Optimal Control Theory and Computer Controller Design* (Cambridge, Mass.: M.I.T. Press, 1968).

B. Widrow, *Adaptive Sampled-Data Systems*, Proc. IFAC, 1, 423-429 (1960).

W. W. Wierwille et al., *Study of Nonstationary Random Process Theory*, NASA-CR-87289 (June 1967).

D. J. Wilde, *Optimum Seeking Methods* (Englewood Cliffs, N. J.: Prentice-Hall Inc., 1964).

D. J. Wilde and C. S. Beightler, *Foundations of Optimization* (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967).

J. W. Wilkinson, *Exact Expressions for the Covariances between Products of Random Variables*, ORNL-TM-1977 (Sept. 1967).

T. J. Williams, *Computers and Process Control*, Ind. Eng. Chem., 58, No. 12, 55-70 (Dec. 1966).

SELECTED CONTROL BIBLIOGRAPHY

S. M. Win, *Time Optimal Control of Non-linear Sampled-Data Systems*, Ph.D. Thesis, Univ. Illinois, 1964. (UM No. 65-935).

A. A. Wolf, *Some Recent Advances in the Analysis and Synthesis of Non-linear Systems*, AIEE Trans. Appl. Ind., 80, No. 57, 289-300 (Nov. 1961).

J. Wolkovitch, *Optimization on Linear and Nonlinear Systems by Minimization of Auxiliary Effort*, STI-TR-900-1 (Feb. 1968).

W. M. Wonham, *Some Applications of Stochastic Differential Equations to Optimal Nonlinear Filtering*, SIAM J. Contr., 2, No. 3, 347-369 (1964).

W. M. Wonham, *Stochastic Problems in Control*, Proc. IBM Sci. Comput. Symp. Contr. Theory Appl. (Yorktown Heights, N. Y., Oct. 1964), 239-248 (1966).

W. M. Wonham, *Optimal Stationary Control of a Linear System with State-Dependent Noise*, J. SIAM Contr., 5, No. 3, 486-500 (Aug. 1967).

S. H. Wu and J. M. Bielefeld, *Frequency Domain Stability Criterion for Discrete Systems with Hysteresis Nonlinearities*, Proc. Hawaii Int. Conf. Syst. Sci. (Honolulu, Jan. 1968), 174-177.

E. E. Yore, *Identification of Dynamic Systems by Digital Computer Modeling in State Space and Component Parameter Identification of Static and Dynamic Systems*, Ph.D. Thesis, U. of Calif., Berkley, 1966. (UM No. 66-15523).

E. E. Yore and Y. Takahashi, *Identification of Dynamic Systems by Digital Computer Modeling in State Space*, J. Basic Eng., 89, No. 2, 295-299 (June 1967).

J. Zaborsky and W. L. Humphrey, *Control without Model or Plant Identification*, Proc. JACC, 366-381 (1964).

J. Zaborsky, *Theoretical and Experimental Research on Digital Adaptive Control System*, NASA-CR-669 (Jan. 1967).

J. Zaborsky, *Continuation of Theoretical and Experimental Research on Digital Adaptive Control System*, NASA-CR-810 (Feb. 1967).

L. A. Zadeh, *An Introduction to State-Space Techniques*, State-Space Techniques for Control Systems Workshop, AIEE (1962).

G. Zames, *Nonlinear Operators for System Analysis*, Ph.D. Thesis, Massachusetts Institute of Technology, 1960.

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